Inflation, Trading Frictions, and Capital Accumulation*

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Abstract

Conventional monetary production models emphasize how inflation erodes real balances and reduces capital's productive return through smaller purchases. We introduce endogenous buyer search intensity into a competitive monetary model with productive capital. We find that the effect of inflation on capital accumulation can be non-monotone, depending on trading frictions. With higher trading frictions, capital can rise at low inflation and fall at higher inflation. Although inflation reduces purchases per meeting, competitive price reductions can raise buyer surplus and encourage more intensive searching efforts. More frequent meetings then give sellers additional opportunities to use capital, potentially offsetting the lower productive return from each monetary trade. At higher inflation, this frequency gain can become too weak to sustain the initial increase in capital. We derive a buyer-surplus condition for that increase near the Friedman rule on an existing regular interior highest-search equilibrium branch. A calibration to U.S. data illustrates how changing trading frictions shifts the balance between these forces. These results suggest that improvements in the ease of trade can alter inflation's effects on productive investment.

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1 Introduction

Inflation is conventionally viewed as discouraging capital accumulation by reducing the scale of monetary trade. In the monetary-production model of Aruoba, Waller, and Wright (2011), capital reduces the effort needed to produce goods sold for money. Inflation lowers real balances and therefore purchases in each monetary meeting. Sellers then produce less, leaving less effort for capital to save. With trading frequency fixed, the lower productive return discourages accumulation. We ask whether this conclusion changes when buyers can search harder and give sellers more opportunities to use capital.

Competitive price reductions can raise the reward from a monetary meeting even as purchases shrink. A smaller purchase lowers marginal production cost and hence the unit price. Near the Friedman rule, the utility lost on the marginal unit is offset by the payment saved on that unit. Buyers also pay less for all the units they retain. This additional price saving raises the surplus from a monetary meeting and encourages search. At higher nominal rates, the consumption loss can outweigh the price saving, weakening the incentive.

The contribution is to connect this buyer search response to the productive return on sellers' capital. We introduce the search choice of Liu, Wang, and Wright (2011) into the price-taking money-and-capital environment of Aruoba et al. (2011). Buyers search for consumption surplus, while sellers hold capital for the effort it saves in production. The equilibrium links these decisions through competitive prices. Capital provides neither payment services nor collateral, so any positive inflation response comes from its productive use. The central question is whether the additional meetings compensate for the smaller saving in each monetary trade.

Our main analytical result gives a buyer-surplus threshold for inflation to raise capital near the Friedman rule. With logarithmic utility and power production costs, the search gain dominates when the surplus supporting the initial search choice is sufficiently small. The price saving then induces a large increase in trading frequency relative to its initial level. The resulting gain in expected effort savings supports more capital despite smaller monetary purchases. This result concerns an existing regular interior equilibrium on the highest-search branch.

Trading frictions determine whether this search response is available. Lower trading frictions can put buyers at the upper bound on their meeting probability. Inflation then cannot generate additional meetings, so capital falls while the bound remains active. Higher trading frictions can leave search interior and allow the frequency effect to offset the size effect. If capital initially rises and the direct search incentive later reverses along a regular interior branch, capital reaches an interior maximum. A unique peak additionally requires a single crossing of the capital derivative.

The capital–output ratio translates the mechanism into a conditional prediction for aggregate data. When capital's share of decentralized output is below its share of centralized output, output rises proportionally more than capital as capital intensity increases. The ratio K/Y therefore falls, reaching a minimum at the capital peak. This share restriction holds in our calibration. Reversing the share ranking reverses the relation between K/Y and capital intensity.

The U.S. calibration illustrates how trading frictions change the capital response. On the computed baseline branch, increased trading frequency initially offsets smaller monetary purchases. Capital reaches a peak when the frequency gain ceases to compensate for the size loss. Stationary welfare peaks later, reflecting the gains and costs of the entire allocation. Reducing the search cost to one fifth of its

calibrated value puts search at its bound over the reported range. Capital then declines with the nominal rate. The Friedman rule has the highest computed stationary welfare in this lower-friction economy.

Payment improvements can change monetary-policy comparisons by removing the scope for inflation to induce more trade. The quantitative welfare comparison is nevertheless conditional. The baseline Friedman-rule allocation remains unverified. Low-rate credit trades also require a specification of participation. Section 4.3 explains these limitations. A policy recommendation would also require a planner solution and an analysis of transitions between stationary allocations.

The data contain patterns resembling the model's conditional predictions, but do not identify the search mechanism. Long-run U.S. and pooled international schedules have shallow interior minima in K/Y. The recent U.S. schedule instead rises, as do schedules in selected countries with greater payment access. Historical shocks and selection on schedule shape affect these comparisons. A reshuffling exercise also generates at least as many U-shaped country schedules as the data. Independent measures of trading costs are needed to assess whether the proposed frequency channel explains the observed patterns.

Section 2 characterizes equilibrium. Section 3 develops the trade-size and trade-frequency mechanism. Section 4 quantifies its implications for capital and welfare. The descriptive evidence follows in Section 5. Section 6 concludes. Proofs and supporting details are in the appendices.

1.1 Related literature

Our mechanism changes the productive return to capital rather than its role in a portfolio. Tobin (1965) obtains a positive inflation effect through substitution between money and capital. Mundell (1963) emphasizes the effect of real balances on saving. A different channel operates in Stockman (1981), where a cash requirement taxes investment purchases. Gomme (1993) studies monetary effects in an endogenous-growth setting. Capital in our economy is purchased in the frictionless market. Inflation instead affects its return through the goods trade that uses it.

Decentralized production is what makes capital sensitive to monetary trade in our framework. Aruoba and Wright (2003) obtain a neoclassical dichotomy when capital produces only in the frictionless market. Aruoba et al. (2011) introduce decentralized production and derive the resulting trade-size channel. Waller (2011) and Chu, Kan, Lai, and Liao (2014) study related growth environments. We retain the price-taking version of Aruoba et al. (2011) and add buyer search. The new frequency margin can reverse the capital response even as monetary purchases shrink.

The frequency adjustment differs from the participation channel in Shi (1999). Participation in that model changes the number of trades. Our household population is fixed. Buyers instead vary search intensity, as on the intensive margin in Liu et al. (2011). We show when this adjustment reverses the capital response. The calibration then evaluates the associated stationary welfare ranking.

Goods-market search provides a different source of non-monotonicity from labor-market frictions. In Gomis-Porqueras, Huangfu, and Sun (2020), changes in the number of active firms interact with changes in capital per firm. Our mechanism operates through the frequency with which sellers use capital in goods trade. Under our factor-share restriction, it also makes K/Y move inversely with capital intensity.

Liquidity services can raise capital demand through a separate channel. Lagos and Rocheteau (2008) allow capital to compete with money as a medium of exchange. Real assets relax liquidity constraints

in Geromichalos, Licari, and Suárez-Lledó (2007). Capital provides collateral in Venkateswaran and Wright (2014). Altermatt and Wipf (2024) examine how a liquidity-driven Mundell–Tobin effect bears on the Friedman rule. We exclude these services to isolate the productive return from more frequent trade.

The buyer's search response belongs to the hot-potato literature. Lagos and Rocheteau (2005), Liu et al. (2011), and Nosal (2011) show how trading institutions shape the incentive to spend money sooner. Shi (1997) and Li (1994) also emphasize trading opportunities. Faster spending can arise from infrequent portfolio adjustment (Ennis, 2009; Chiu and Molico, 2010). Other sources include price-comparison search (Head and Kumar, 2005; Wang, 2016) and participation or entry (Huangfu, 2018; Hiraguchi and Kobayashi, 2025). Hu and Zhang (2019) analyze responses to the inflation tax and implementable allocations. Our contribution traces the response from competitive prices through buyer surplus to capital use.

Information and intermediation frictions can also generate non-monotone inflation effects. Azariadis and Smith (1996), Choi, Smith, and Boyd (1996), and Huybens and Smith (1999) study these mechanisms. Boyd, Levine, and Smith (2001) document links between inflation and financial-sector performance. Our economy has no banks or collateral constraints. This allows us to isolate the interaction between the size and frequency of goods trade.

2 Model

The model gives buyers a search choice that changes how often sellers use capital. We add this margin to the price-taking economy of Aruoba et al. (2011). Capital produces in both the centralized and decentralized markets. Money is needed only for anonymous decentralized trade. Households choose their portfolios before learning whether they will buy or sell, so anticipated trading opportunities affect ex ante capital demand.

2.1 Environment and timing

Each period consists of a decentralized market (DM) followed by a centralized market (CM). A unit measure of infinitely lived households discounts between periods by $\beta \in (0,1)$. There is no discounting within a period. The CM is frictionless: households consume a general good, supply labor, settle debts, and choose next-period money and capital. Competitive firms produce the general good. It can be transformed one for one into capital. The DM instead involves a nonstorable specialized good traded subject to matching and payment frictions.

Households choose portfolios without knowing their next DM role. At the start of each DM, a household becomes a buyer with probability $n \in (0,1)$. Otherwise it becomes a seller. Roles are drawn independently over time. Buyers value the specialized good but cannot produce it. Sellers can produce it but do not value its consumption. A seller uses its own capital in DM production. A buyer may spend money there, but its capital produces only in the CM that period. Sellers carry their money into the CM.

Buyer search determines how often DM trade occurs. A buyer chooses a meeting probability $\sigma \in [0, \bar{\sigma}]$ at utility cost $\mu(\sigma)$, where $\bar{\sigma} = \min\{1, (1-n)/n\}$. We assume $\mu(0) = \mu'(0) = 0$. For $\sigma > 0$,

marginal cost is positive and strictly increasing: $\mu'(\sigma) > 0$ and $\mu''(\sigma) > 0$. Each seller serves at most one buyer. If all buyers choose σ, a seller trades with probability $n\sigma/(1-n)$. The search bound keeps this probability at most one. Our analytical and quantitative exercises use $\mu(\sigma) = \nu\sigma^2/2$, with $\nu > 0$.

Monitoring determines whether a meeting can be settled with credit. A meeting is monitored with probability $\rho \in [0, 1)$, independently of search intensity. Monitoring records the buyer's identity and promised payment. This permits costless enforcement of repayment in the following CM. Anonymous meetings lack this enforcement, so payment requires fiat money. Capital and claims on capital cannot serve as money or collateral. The economy has no banks. Buyers and sellers take prices as given in separate markets for monitored and anonymous trade.

Capital lowers sellers' DM effort costs in addition to producing CM output. Let x denote household CM consumption and h denote hours. A buyer consumes q in a DM trade. A seller supplies production effort e. The indicator $\mathbb{I}$ equals one for a buyer and zero for a seller. Before subtracting search cost, the payoff from CM activity and a realized DM trade is

$$\underbrace{U(x) - \bar{A}h}_{\text{CM}} + \underbrace{\mathbb{I}\, u(q) - (1 - \mathbb{I})\, e}_{\text{DM}}, \tag{2.1}$$

where $\bar{A} > 0$, and U and u are increasing and strictly concave. DM utility and production effort are realized only when trade occurs. A buyer pays the search cost whether or not it meets a seller. A seller produces $q = f(e, k)$ with its own capital k. Inverting this technology gives effort cost $c(q, k)$, with $c_q > 0$, $c_{qq} > 0$, $c_k < 0$, $c_{kk} > 0$, and $c_{qk} < 0$. CM firms produce with a constant-returns technology $F(K, H)$, where K and H are aggregate capital and labor. Capital depreciates at rate $\delta \in (0, 1)$.

Monetary policy affects trade by changing the opportunity cost of holding money. The money supply grows at rate τ through lump-sum transfers. Stationary inflation equals money growth, and the Fisher relation is $1 + i = (1 + \tau)/\beta$. Thus a higher nominal rate i represents higher inflation with the discount factor held fixed. We consider $i \geq 0$, with $i = 0$ denoting the Friedman rule. Money balances m are measured in units of the CM good. Primes denote next-period choices, and time subscripts are suppressed. Superscripts c and m denote credit and monetary trades, respectively. Figure 1 summarizes the timing.

2.2 The centralized market

Competitive CM firms pay factors their marginal products. Given rental rate r and wage w, a firm solves $\max_{K,H\geq 0}\{F(K, H) - rK - wH\}$, yielding $r = F_K(K, H)$ and $w = F_H(K, H)$. Constant returns imply zero profits. All households rent their capital to CM firms, regardless of their role in the preceding DM.

Quasilinear labor disutility makes the household's CM value linear in wealth. Let b be the debt due from a monitored purchase, T the lump-sum transfer, and V the value at the start of the next DM. The CM value is

$$W(m, b, k) = \max_{(x,h,m',k')\in\mathbb{R}^4_+} \left\{ U(x) - \bar{A}h + \beta V(m', k') \;\middle|\; x + (1+\tau)m' + k' = m - b + (1 + r - \delta)k + wh + T \right\}. \tag{2.2}$$

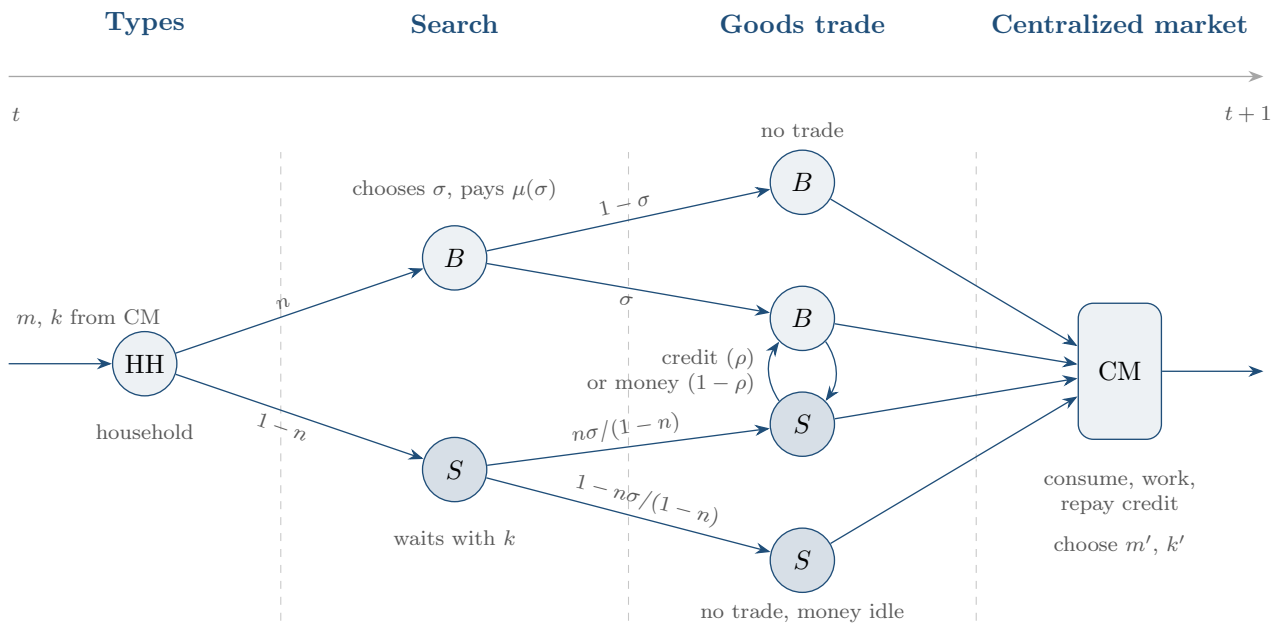


Figure 1: Timing within a period. A household (HH) becomes a buyer (B) or a seller (S). A buyer who finds a trade buys on credit if the trade is monitored and pays with money otherwise. All households then enter the CM, where credit is repaid and portfolios for the next period are chosen.

Purchasing next-period real balances m' costs $(1+\tau)m'$ today. With interior labor supply, eliminating h gives the first-order and envelope conditions

$$U'(x) = \frac{\bar{A}}{w}, \qquad \frac{\bar{A}(1+\tau)}{w} = \beta V_m(m', k'), \qquad \frac{\bar{A}}{w} = \beta V_k(m', k'), \tag{2.3}$$

$$W_m = \frac{\bar{A}}{w}, \qquad W_b = -\frac{\bar{A}}{w}, \qquad W_k = \frac{(1+r-\delta)\bar{A}}{w}. \tag{2.4}$$

The CM converts asset payoffs into a common utility value through the labor margin. An extra unit of wealth saves $\bar{A}/w$ in labor disutility. The portfolio conditions in (2.3) therefore compare today's acquisition cost with the discounted marginal value of each asset in the next DM. Money is costly to carry because its acquisition cost includes the growth of the price level. Capital is costly to acquire but earns a productive return. Appendix A.7.1 derives the conditions by eliminating labor from the budget constraint.

Interior labor supply removes the effect of incoming wealth on portfolio choices. Households absorb the gains and losses from earlier trade by adjusting hours. With a common portfolio choice, they therefore enter the next DM with identical money and capital holdings, as in Lagos and Wright (2005). This property keeps the stationary equilibrium from requiring a distribution of wealth. We set individual capital $k = K$ in symmetric stationary equilibrium.

2.3 The decentralized market

A household enters the DM before it learns whether it will buy or sell, so its value at the start of the DM is the expected value over the two types,

$$V(m,k) = nV^b(m,k) + (1-n)V^s(m,k), \tag{2.5}$$

where V^b and V^s are the values of a buyer and of a seller. We derive them backward, starting with trade in the two markets and ending with the search decision.

Monitored and anonymous markets. Let p^c be the price of the DM good in a monitored trade, in units of the CM good. A buyer who buys q on credit enters the CM with the debt $p^c q$, and a seller who sells q receives $p^c q$ there. Taking p^c as given, the buyer and the seller solve

$$\max_{q\geq 0}\{u(q) + W(m, p^c q, k)\} \qquad \text{and} \qquad \max_{q\geq 0}\{-c(q,k) + W(m + p^c q, 0, k)\}.$$

Because W is linear, with $W_b = -\bar{A}/w$ and $W_m = \bar{A}/w$, the first-order conditions are $u'(q) = \bar{A}p^c/w$ for the buyer and $c_q(q,k) = \bar{A}p^c/w$ for the seller. Market clearing requires the quantity bought to equal the quantity sold, so

$$u'(q^c) = \frac{\bar{A}p^c}{w} = c_q(q^c, k). \tag{2.6}$$

Both problems are strictly concave, so Equation (2.6) determines q^c uniquely. With credit the buyer consumes the quantity at which marginal utility equals marginal cost, which is the quantity a planner would choose given the seller's capital; the debt $b = p^c q^c$ is repaid in the CM that follows.

In an anonymous meeting the buyer must pay with money, so it can spend at most its balances m. Let p be the price of the DM good in an anonymous trade, in units of the CM good. Taking p as given, the buyer and the seller solve

$$\max_{q\geq 0}\{u(q) + W(m - pq, 0, k) \mid pq \leq m\} \qquad \text{and} \qquad \max_{q\geq 0}\{-c(q,k) + W(m + pq, 0, k)\}.$$

The seller's first-order condition is $c_q(q,k) = \bar{A}p/w$, as in a monitored trade. The buyer's problem differs because of the cash constraint. When $i > 0$, money is costly to hold, so a household carries no more money than it would spend in a trade and the constraint binds: $q^m = m/p$, and market clearing gives $c_q(q^m, k) = \bar{A}p/w$. At this quantity the buyer would like to buy more, $u'(q^m) \geq \bar{A}p/w$, with equality only at $i = 0$, and the size of the gap is set by the choice of money balances derived below. This gap is how inflation acts on DM trade: it keeps the monetary quantity q^m below the credit quantity q^c.

Buyers and search. A buyer chooses search intensity knowing what it gains from each type of trade. Its surplus from a monitored and from an anonymous trade are

$$\begin{aligned} S^c &= u(q^c) - \frac{\bar{A}p^c}{w}q^c = u(q^c) - q^c\, c_q(q^c, k), \\ S^m &= u(q^m) - \frac{\bar{A}p}{w}q^m = u(q^m) - q^m\, c_q(q^m, k), \end{aligned} \tag{2.7}$$

where the second equalities use Equation (2.6) and the seller's condition in the anonymous market.

Each surplus is the utility from the goods bought minus the utility value of the payment. Because the price equals the seller's marginal cost, the buyer pays marginal cost on every unit, so the payment $q\,c_q(q,k)$ exceeds the seller's cost $c(q,k)$ and the gains from trade are split: the seller keeps $q\,c_q - c$ and the buyer keeps S. The two surpluses differ only through the quantity traded. In a monitored trade the buyer buys until marginal utility equals marginal cost. In an anonymous trade the cash constraint holds the quantity down, which lowers both the utility the buyer receives and the price it pays on every unit.

A buyer that searches with intensity σ finds a trade with probability σ, so its value is

$$V^b(m,k) = W(m,0,k) + \max_{\sigma\in[0,\bar{\sigma}]}\Big\{-\mu(\sigma) + \sigma\big[\rho S^c + (1-\rho)S^m\big]\Big\}. \tag{2.8}$$

Search buys a chance at the expected surplus $\rho S^c + (1-\rho)S^m$ at the utility cost $\mu(\sigma)$; a buyer that finds no trade simply carries its money into the CM.

At an interior optimum, search intensity equates the marginal cost of search to the expected surplus from a trade:

$$\mu'(\sigma) = \rho S^c + (1-\rho)S^m. \tag{2.9}$$

Because $\mu'(0) = 0$ and μ is strictly convex, a positive expected surplus yields a unique positive optimum. It solves Equation (2.9) when interior; at $\sigma = \bar{\sigma}$, optimality instead requires $\rho S^c + (1-\rho)S^m \geq \mu'(\bar{\sigma})$. Buyers search harder when trades are worth more to them, and the two surpluses respond differently to monetary policy. The monitored surplus S^c depends only on q^c, which Equation (2.6) ties to the seller's capital, so for given capital it does not depend on i. The anonymous surplus S^m depends on q^m, which the cash constraint links to the cost of holding money. Inflation therefore affects search only through the surplus from anonymous trade, and Section 3 shows that this surplus can rise with i.

Sellers. A seller's value at the start of the DM depends on how often it meets a buyer. It takes the buyers' search intensity σ as given and trades with probability $n\sigma/(1-n)$; the trade is monitored with probability ρ and anonymous otherwise. Using the linearity of W, its value is

$$V^s(m,k) = W(m,0,k) + \frac{n\sigma}{1-n}\Big\{\rho\Big[\frac{\bar{A}p^c}{w}q^c - c(q^c,k)\Big] + (1-\rho)\Big[\frac{\bar{A}p}{w}q^m - c(q^m,k)\Big]\Big\}. \tag{2.10}$$

A seller keeps its money until the CM, so its balances enter only through W. Each term in braces is the seller's profit from a trade, the value of the payment minus the effort cost, and capital raises both profits by lowering that cost. Because the seller trades more often when buyers search harder, search raises the value of the capital a household brings into the DM. This link between search and the return to capital is the new element of the model.

Marginal values of money and capital. Money and capital are chosen before the type is known, so their marginal values follow from the expected value in Equation (2.5). An extra unit of money is worth $\bar{A}/w$ in the CM to every household, and a buyer in an anonymous trade can also spend it on the DM good, since $q^m = m/p$. Differentiating Equations (2.10) and (2.8) and using $c_q(q^m,k) = \bar{A}p/w$

gives

$$V_m(m,k) = \frac{\bar{A}}{w}\Big[1 + \underbrace{n\sigma(1-\rho)\Big(\frac{u'(q^m)}{c_q(q^m,k)} - 1\Big)}_{\text{liquidity premium}}\Big]. \tag{2.11}$$

A unit of money is worth its CM value plus a liquidity premium. The premium is earned only when the money is actually spent, which happens with probability $n\sigma(1-\rho)$: the money of a seller, of a buyer that finds no trade and of a buyer that trades on credit stays idle until the CM. When it is spent, it buys goods whose utility exceeds their cost in proportion to $u'(q^m)/c_q(q^m,k) - 1$. Search intensity enters as the frequency of spending: the harder buyers search, the more often a unit of money finds a trade, and the more it is worth to hold. With lower trading frictions, buyers search harder, so money is spent more often and its liquidity premium rises for a given size of trade.

Capital earns a return in both markets. Every household rents its capital to CM firms, and a seller also uses it to produce in the DM. Because the seller takes prices as given, a marginal unit of capital changes its profit only by lowering the effort cost, by $-c_k$. A household is a seller with probability $1-n$ and a seller trades with probability $n\sigma/(1-n)$, so differentiating Equations (2.10) and (2.8) gives

$$V_k(m,k) = \underbrace{\frac{\bar{A}}{w}(1+r-\delta)}_{\text{CM return}} + \underbrace{n\sigma\big[\rho\big(-c_k(q^c,k)\big) + (1-\rho)\big(-c_k(q^m,k)\big)\big]}_{\text{DM cost saving}}. \tag{2.12}$$

A unit of capital is worth its CM return plus a DM cost saving. The CM return, the rental income net of depreciation valued at $\bar{A}/w$, is earned by every household. The DM cost saving is earned only when the household sells and meets a buyer, which happens with probability $n\sigma$; in each such sale capital saves the seller the effort $-c_k$, and the saving is larger the larger the trade, since producing more takes more effort for capital to save. Search intensity enters as the frequency with which capital is used in the DM: the harder buyers search, the more often sellers meet them, and the more often each unit of capital saves effort. With lower trading frictions, buyers search harder, so capital is used in the DM more often and its DM return rises for a given size of trade.

2.4 Stationary monetary equilibrium

The money Euler equation balances a certain holding cost against a return earned only in monetary trade. Substituting (2.11) into (2.3) and using the Fisher relation gives

$$i = \underbrace{n\sigma(1-\rho)}_{\text{probability of use}} \underbrace{\Big(\frac{u'(q^m)}{c_q(q^m,K)} - 1\Big)}_{\text{gain per use}}. \tag{2.13}$$

Money must deliver a larger liquidity gain per use when its holding cost rises. At fixed capital and search, buyers obtain that larger marginal gain by carrying less money and purchasing less in an anonymous meeting. Higher search works in the opposite direction: more frequent spending reduces the liquidity gain needed per use to cover a given holding cost. Equation (2.13) therefore connects monetary purchases to both policy and trading frequency. At $i = 0$, the liquidity wedge vanishes and $q^m = q^c$.

The capital Euler equation separates the frequency of use from the saving in each use. Substituting (2.12) into the capital condition in (2.3) gives

$$\frac{1}{\beta} - 1 + \delta = r + \underbrace{n\sigma}_{\text{probability of use}} \underbrace{\frac{w}{\bar{A}}[\rho(-c_k(q^c, K)) + (1-\rho)(-c_k(q^m, K))]}_{\text{saving per use}}. \tag{2.14}$$

Capital demand depends on both the frequency of production and the effort saved in each trade. The expected DM saving supplements the CM rental return to cover impatience and depreciation. Smaller monetary purchases reduce the saving per meeting. More search increases the number of meetings and also supports larger monetary purchases through (2.13). After either margin changes, capital adjusts until the combined productive return again equals the required return in (2.14). The equilibrium comparison thus concerns how much capital can be supported at that return.

Search and capital must be solved jointly because each changes the return to the other. Given these two choices, the trade and CM conditions determine quantities, prices, labor, and consumption. The CM resource constraint includes consumption and replacement investment. Credit repayments and money transfers redistribute purchasing power without using additional resources.

Definition 1. Given $i \geq 0$, a stationary monetary equilibrium with positive search is an allocation satisfying the following conditions, with quantities (q^c, q^m), search σ, capital K, labor H, and consumption x:

(i) Monitored trade: (2.6), with $k = K$.

(ii) Money demand: (2.13).

(iii) Search: (2.9) if $0 < \sigma < \bar{\sigma}$, and $\rho S^c + (1-\rho)S^m \geq \mu'(\bar{\sigma})$ if $\sigma = \bar{\sigma}$.

(iv) Capital: (2.14).

(v) Centralized market: $r = F_K(K, H)$, $w = F_H(K, H)$, $U'(x) = \bar{A}/w$, and $F(K, H) = x + \delta K$.

Prices satisfy $\bar{A}p^c/w = u'(q^c)$ and $\bar{A}p/w = c_q(q^m, K)$. Real balances are $m = pq^m$ for $i > 0$, with the minimum-balance convention at $i = 0$, and $T = \tau m$.

The complementarity between aggregate capital and search can support multiple stationary monetary equilibria. More search increases the frequency with which capital saves sellers effort. More capital lowers production costs and supports larger trades, raising buyer surplus under the logarithmic specification below. This aggregate feedback operates through market prices, which individual households take as given. Let $\Phi(\sigma)$ be marginal search cost minus expected buyer surplus after capital satisfies its Euler equation. The following proposition gives a sufficient condition for multiplicity under the functional forms of Section 3.

Proposition 1. *Under the functional forms of Section 3, with logarithmic DM utility and $i > 0$, suppose $\Phi(\sigma_1) < 0$ for some $\sigma_1 \in (0, \bar{\sigma})$. Then at least two stationary monetary equilibria have positive search: one with $\sigma \in (0, \sigma_1)$ and another with $\sigma \in (\sigma_1, \bar{\sigma}]$.*

Proof. See Appendix A.1.

We select the equilibrium with the highest search intensity whenever several stationary monetary equilibria exist. At an interior solution, this is the largest zero of the reduced search condition Φ. The selection keeps the comparative statics on a specified branch; it establishes neither uniqueness nor dynamic stability. Local derivatives additionally require that branch to be differentiable and regular.

3 Trade size, trade frequency, and capital

An increase in trading frequency raises capital only when it offsets the smaller production saving in each monetary trade. Competitive price reductions can make a meeting more attractive to buyers and encourage search. The capital Euler equation determines whether the resulting meetings support more accumulation. The surplus threshold below connects these two decisions near the Friedman rule. The full derivative then traces their balance as the nominal rate rises.

3.1 Search incentives and the return to capital

Logarithmic utility makes the size and frequency effects explicit. Set $u(q) = \bar{C}\ln q$ for DM utility and $U(x) = \bar{B}\ln x$ for CM utility. Production costs are $c(q,k) = q^{\omega}k^{1-\omega}$. CM production is $F(K,H) = K^{\alpha}H^{1-\alpha}$. Search costs are $\mu(\sigma) = \nu\sigma^2/2$. We require $\bar{B}, \bar{C}, \nu > 0$, $\alpha \in (0,1)$, and $\omega > 1$. Define capital intensity as $\hat{k} = K/H$. The DM capital share is $\zeta = (\omega-1)/\omega$. The CM conditions imply

$$K(\hat{k}) = \frac{\bar{B}(1-\alpha)\hat{k}}{\bar{A}[1-\delta\hat{k}^{1-\alpha}]}, \qquad \mathcal{H}(\hat{k}) = \hat{k}^{\alpha-1} - \delta > 0,$$

where $\mathcal{H}$ is CM output net of depreciation per unit of capital, distinct from labor H. Section 4 retains these technologies but uses constant-relative-risk-aversion DM utility.

The search curve equates the cost of another trading opportunity with the expected buyer surplus it delivers. Its cost term is $\nu\sigma$, the marginal disutility of search. Its benefit term averages the gains from credit and monetary meetings. Substituting the trade quantities into (2.9) gives, for interior search,

$$\mathcal{T}(\hat{k},\sigma;i) = \nu\sigma - \bar{C}\{\rho[\ln q^c(\hat{k}) - 1] + (1-\rho)[\ln q^m(\hat{k},\sigma;i) - \gamma(\sigma,i)]\} = 0, \tag{3.1}$$

where

$$q^c(\hat{k}) = \left(\frac{\bar{C}}{\omega}\right)^{1/\omega} K(\hat{k})^{\zeta}, \quad q^m = q^c\gamma^{1/\omega}, \quad \gamma(\sigma,i) = \frac{n\sigma(1-\rho)}{n\sigma(1-\rho)+i}.$$

The variable γ measures the monetary distortion within a meeting. It is the ratio of marginal production cost to marginal utility in anonymous trade. At the Friedman rule, $\gamma = 1$ and monetary purchases equal credit purchases. A higher nominal rate lowers γ at fixed search, reducing q^m relative to q^c. More search raises γ by spreading the holding cost over more monetary meetings. Capital increases both quantities by lowering production costs. At the search bound, the expected surplus may still exceed marginal search cost; the constrained search condition is then $\mathcal{T} \leq 0$.

The capital curve combines the number of meetings with the saving in an average meeting. Define $\mathcal{P}(\sigma, i) = \rho + (1-\rho)\gamma(\sigma, i)$. Equation (2.14) becomes

$$J(\hat{k}, \sigma; i) = \frac{1}{\beta} - \left[1 + \alpha \hat{k}^{\alpha-1} - \delta + n\sigma \mathcal{P}(\sigma, i) \frac{\zeta \bar{C}}{\bar{B}} \mathcal{H}(\hat{k})\right] = 0. \tag{3.2}$$

The weight $\mathcal{P}$ measures how the composition of trade affects capital's DM return. A credit meeting receives weight one because it has quantity q^c. A monetary meeting receives weight γ because its smaller quantity leaves less effort for capital to save. With power costs, the saving is proportional to q^ω, and $(q^m/q^c)^\omega = \gamma$. The product $n\sigma\mathcal{P}$ therefore combines meeting frequency with the relative production saving in the two types of trade. At fixed search, a higher nominal rate lowers this product through γ. Capital intensity must then fall to restore $J = 0$, since $J_{\hat{k}} > 0$. Subscripts on T and J below denote partial derivatives of $\mathcal{T}$ and J, respectively.

3.2 Why smaller trades can induce more search

The gain in buyer surplus near the Friedman rule comes from a lower price on the units still consumed. At $i = 0$, marginal utility equals the competitive price in utility units. For a small reduction in purchases, the utility lost on the marginal unit is therefore offset to first order by the payment saved on that unit. The lower quantity also reduces marginal production cost, so the competitive price falls on every retained unit. This additional saving raises the reward from a monetary meeting at fixed capital. An interior buyer responds by searching more. The response is to a change in the market price; no individual buyer perceives its own quantity choice as changing that price.

Lemma 1. *At an interior equilibrium with $i = 0$, $\gamma = 1$ and $q^m = q^c$. The partial derivatives satisfy*

$$T_i = -\frac{\zeta \bar{C}}{n\sigma} < 0, \qquad T_\sigma = \nu > 0, \qquad J_i = \frac{\zeta \bar{C}}{\bar{B}} \mathcal{H}(\hat{k}) > 0, \qquad J_{\hat{k}} > 0.$$

Thus the search curve calls for more search at fixed capital, $-T_i/T_\sigma > 0$, while the capital curve calls for less capital at fixed search, $-J_i/J_{\hat{k}} < 0$.

Proof. See Appendix A.2.

The price incentive weakens as the cash constraint pushes consumption further below the credit quantity. For a marginal reduction in q^m, the utility loss net of the payment saved on the marginal unit is $u'(q^m) - c_q(q^m, K)$. The price saving on retained units is $q^m c_{qq}(q^m, K)$. The first term is zero at the Friedman rule and positive at $i > 0$. Buyer surplus rises as purchases fall only if the second term exceeds the first. For power production costs, this comparison at fixed capital and search is

$$\frac{i}{n\sigma(1-\rho)} < \omega - 1.$$

The condition compares the holding cost per monetary use with the elasticity of marginal production cost. More frequent spending lowers the left side, while a steeper marginal-cost schedule raises the right side. Both make the price saving more likely to dominate. Appendix A.3 derives the comparison for general costs. This direct price incentive is only one component of the equilibrium search response, which also depends on capital's adjustment.

3.3 When the frequency effect raises capital

The local capital response depends on how strongly search reacts relative to the initial frequency of trade. Let $\kappa(\sigma, i)$ solve the capital condition $J = 0$ at given search and policy. At the Friedman rule, expected buyer surplus after this capital adjustment is $s(\sigma) = \bar{C}[\ln q^c(\kappa(\sigma, 0)) - 1]$. Write selected search there as σ^{FR}. The cost threshold $\nu_c = s(\bar{\sigma})/\bar{\sigma}$ determines whether search is at its bound. Derivatives at $i = 0$ are right derivatives. An interior equilibrium is regular when the Jacobian of $(\mathcal{T}, J)$ with respect to $(\hat{k}, \sigma)$ is nonsingular.

Lemma 2. *Consider an existing positive highest-search equilibrium at $i = 0$, regular if search is interior.*

(a) If $\nu < \nu_c$, search remains at $\bar{\sigma}$ for sufficiently small i and $d\hat{k}/di|_{i=0} < 0$.

(b) If $\nu > \nu_c$, search is interior and

$$\begin{aligned}
\frac{d\hat{k}}{di}\Big|_{i=0} > 0 \quad &\Longleftrightarrow \quad s(\sigma^{FR}) < \zeta\bar{C}, \\
\frac{d\hat{k}}{di}\Big|_{i=0} = 0 \quad &\Longleftrightarrow \quad s(\sigma^{FR}) = \zeta\bar{C}, \\
\frac{d\hat{k}}{di}\Big|_{i=0} < 0 \quad &\Longleftrightarrow \quad s(\sigma^{FR}) > \zeta\bar{C}.
\end{aligned}$$

(c) Among costs supporting positive interior highest-search equilibria, $s(\sigma^{FR})$ strictly decreases with ν. On a connected cost interval with a continuous selected branch, any crossing of $s(\sigma^{FR}) = \zeta\bar{C}$ is unique. Denote its cost by $\hat{\nu}$; the local capital response is negative below the crossing and positive above it. If $s(\bar{\sigma}) \leq \zeta\bar{C}$, it is positive at every existing interior regular equilibrium with $\nu > \nu_c$; if $s(0) \geq \zeta\bar{C}$, it is negative at every such equilibrium.

(d) If $\zeta < \alpha$, then $d(K/Y)/d\hat{k} < 0$.

Proof. See Appendix A.4.

A small initial buyer surplus makes the induced search gain large relative to the direct loss in capital's return. At the Friedman rule, the interior search condition is $\nu\sigma^{FR} = s(\sigma^{FR})$. Holding capital fixed, Lemma 1 gives a search response of $-T_i/T_\sigma$. Its contribution to the capital return is $-J_\sigma(-T_i/T_\sigma)$, while the direct loss from smaller monetary purchases is J_i. The ratio of the gain to the loss is $\zeta\bar{C}/s(\sigma^{FR})$. When this ratio exceeds one, the additional meetings more than compensate for the smaller saving per meeting. The regular highest-search selection preserves this sign after capital and search adjust jointly.

Higher trading frictions can make this proportional search response stronger without raising the level of capital. On the selected interior branch, a higher search cost lowers both search and capital. The resulting smaller trades reduce the surplus supporting the initial search choice. The ratio $\zeta\bar{C}/s(\sigma^{FR})$ consequently rises wherever the selected positive-search equilibrium exists. This explains why a higher-friction economy can have a positive local inflation response even though a lower-friction economy holds more capital at a given nominal rate. At the search bound, the frequency response is unavailable and capital falls. The threshold $\nu = \nu_c$ requires a separate check of the switch between interior and constrained search.

The relation between capital and K/Y depends on the capital shares in the two production sectors. Total output is $Y = F(K, H) + Y^{DM}$, with DM output valued at transaction prices in CM goods. The

power cost function assigns capital the share ζ of DM output, while its CM share is α. The capital Euler equation weights each sector's output by its capital share, giving

$$\frac{K}{Y} = \frac{\zeta}{1/\beta - 1 + \delta - (\alpha - \zeta)\hat{k}^{\alpha-1}}. \tag{3.3}$$

When $\zeta < \alpha$, output rises proportionally more than capital as capital intensity increases. The CM rental return falls with $\hat{k}$, so the DM must contribute more to the fixed required return on capital. Because the DM has the lower capital share, each unit of capital income there corresponds to more output. The shift in the source of capital income therefore raises output per unit of capital and lowers K/Y. The restriction, equivalently $\omega < 1/(1-\alpha)$, holds in the calibration. Equal shares leave the ratio constant, and the reverse ranking reverses its response. The identity does not require logarithmic DM utility; Appendix A.7.1 gives the derivation.

3.4 From a local response to a capital peak

An initial increase in capital leads to a peak only if the frequency gain later becomes too weak to offset the size loss. To follow that balance along an interior branch, let γ_i denote the partial derivative at fixed search. Define $D = T_{\hat{k}}J_\sigma - T_\sigma J_{\hat{k}}$, $N = T_\sigma J_i - J_\sigma T_i$, and $M = J_{\hat{k}}T_i - T_{\hat{k}}J_i$. The next lemma expresses the allocation derivatives in terms of these objects.

Lemma 3. *On a differentiable, interior, regular branch of highest-search equilibria, $d\hat{k}/di = N/D$ and $d\sigma/di = M/D$, with $D < 0$ and*

$$N = n\frac{\zeta\bar{C}}{\bar{B}}\mathcal{H}(\hat{k})[-(1-\rho)\gamma_i]\mathcal{N}, \qquad \mathcal{N} = \underbrace{\sigma\nu}_{\text{trade-size term}} + \underbrace{\bar{C}\mathcal{P}\Big(\frac{1}{\omega\gamma} - 1\Big)}_{\text{trade-frequency term}}.$$

Hence capital intensity rises when $\mathcal{N} < 0$, falls when $\mathcal{N} > 0$, and has zero derivative when $\mathcal{N} = 0$. If $i \geq (\omega - 1)n\sigma(1-\rho)$, then $\mathcal{N} > 0$ and capital intensity falls. At fixed i, lowering ν weakly raises selected search and capital intensity, strictly if the initial selected equilibrium is interior. Under $\zeta < \alpha$, it weakly lowers K/Y.

Proof. See Appendix A.5.

The numerator separates the loss from smaller trades from the gain induced through buyer search. The trade-size term in $\mathcal{N}$ is positive. The trade-frequency term is negative while $\gamma > 1/\omega$, reflecting the direct price incentive established above. Capital rises only if this negative term dominates, since $D < 0$ on the regular highest-search branch. At the Friedman rule, the condition reduces to the buyer-surplus threshold in Lemma 2. Once $\gamma \leq 1/\omega$, the search term provides no offset and capital falls. A peak can occur before that point: a positive search incentive need not be strong enough to sustain accumulation.

The highest-search selection fixes the sign of the derivative denominator. After capital satisfies $J = 0$, the reduced search condition is marginal cost minus expected surplus. At its largest regular interior zero, this difference crosses zero from below. The identity $\Phi'(\sigma) = -D/J_{\hat{k}}$ therefore gives $D < 0$ because $J_{\hat{k}} > 0$. Feedback changes the magnitude of the capital response, while the numerator determines its sign on this branch.

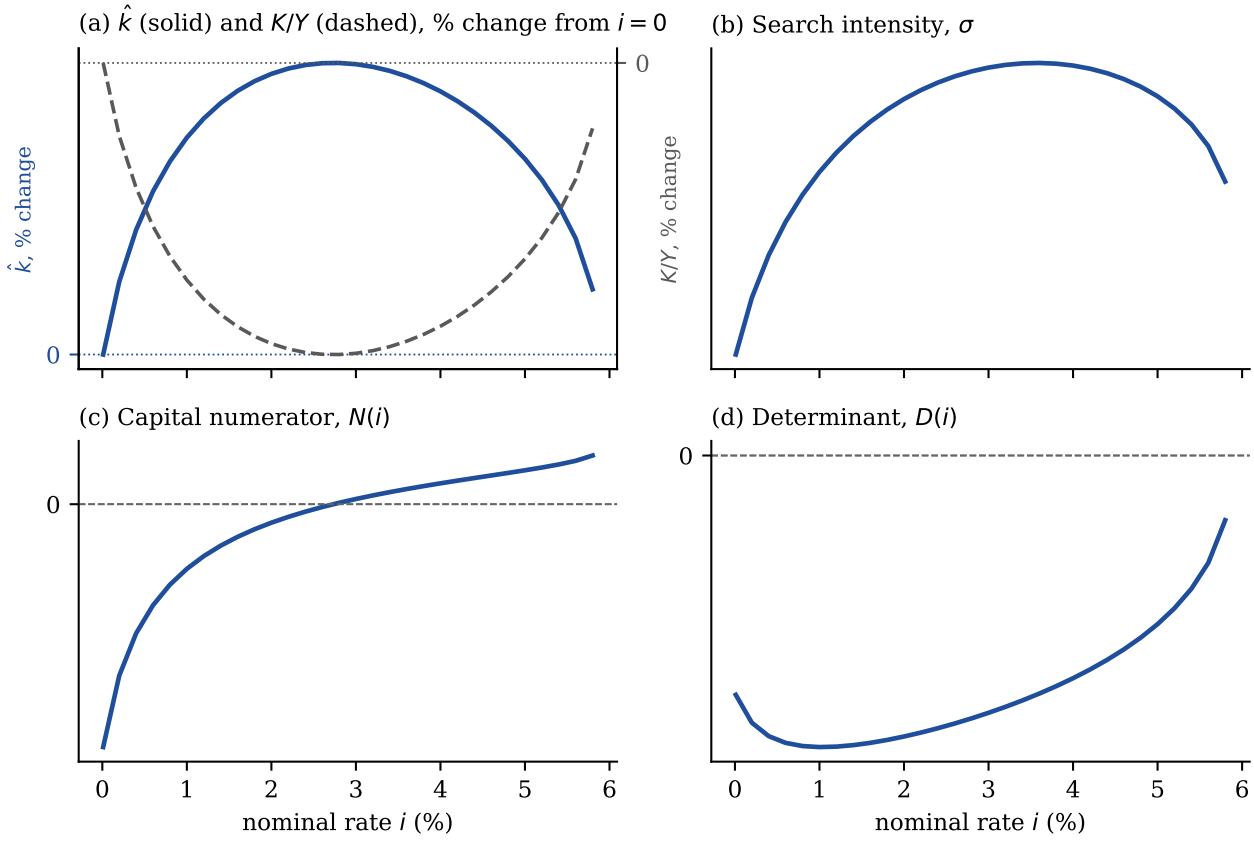


Figure 2: Capital peak and derivative balance. The highest-search equilibrium has an interior peak in capital intensity and a minimum in K/Y. Panels show the allocation, search, the derivative numerator N, and the determinant D. Parameters: $\beta = 0.96$, $\alpha = 0.33$, $\delta = 0.07$, $\rho = n = 0.5$, $\bar{A} = \bar{C} = 1$, $\omega = 1.45$, $\bar{B} = 15$, and $\nu = 0.2$.

Proposition 2. *Follow an existing highest-search branch on $I = [0, \bar{i}]$, where $\bar{i} > 0$.*

(a) If the branch is interior and regular on I, $s(\sigma^{FR}) < \zeta\bar{C}$, and $i_h \geq (\omega - 1)n\sigma(i_h)(1 - \rho)$ for some $i_h \in I$, then $\hat{k}'(0) > 0$ and $\hat{k}'(i_h) < 0$. Thus $\hat{k}$ has an interior maximum on $[0, i_h]$. If N changes sign exactly once on I, that maximum is unique.

(b) If search remains at $\bar{\sigma}$ throughout I, then $\hat{k}'(i) < 0$ throughout the interval.

(c) Under $\zeta < \alpha$, each maximum in part (a) is a minimum of K/Y, and K/Y increases throughout the interval in part (b).

Proof. See Appendix A.6.

Search is still rising at an interior capital peak under the analytical functional forms. Differentiating $J = 0$ at $\hat{k}' = 0$ gives $d\sigma/di = -J_i/J_\sigma > 0$. The search response at that point exactly offsets the direct loss in capital's DM return. Beyond the peak, search can continue rising but no longer generates enough production savings to support further accumulation. Figure 2 illustrates this distinction: N crosses zero once while D remains negative, and search peaks after capital. Under the factor-share restriction, the minimum in K/Y occurs at the capital peak.

The declining-capital example shows why faster trading alone is insufficient for accumulation. In Figure 8, a larger CM utility weight supports larger trades and a higher initial buyer surplus. The proportional search response is therefore weaker. Search initially rises, but the derivative numerator N stays positive along the plotted branch. With $D < 0$, capital intensity declines throughout. The example distinguishes the incentive to trade more often from the strength of the response needed to raise capital.

The search bound provides the clearest case in which inflation reduces capital. When lower trading frictions keep buyers at that bound, a higher nominal rate cannot create another meeting. It only reduces monetary purchases and the production saving associated with them, recovering the trade-size channel of Aruoba et al. (2011). With interior search, the buyer-surplus threshold instead determines the initial capital response. The derivative balance determines how that response changes at higher rates. We next examine these margins numerically using a calibration that allows DM utility curvature to differ from the logarithmic analytical case.

4 Trading frictions, capital, and welfare

The calibration shows how the same monetary-policy change can produce different capital paths at different trading costs. The computed baseline branch has interior capital and welfare peaks. Reducing the search cost puts buyers at their search bound, after which capital and welfare decline with the nominal rate. These are numerical results for CRRA utility, rather than applications of every logarithmic-utility result above.

4.1 Calibration

The calibration matches annual U.S. money demand and production moments for 1980–2007. Payment-cost and payment-share measures are unavailable for that full sample, so their proxies come from outside it. DM utility is $u(q) = \bar{C}(q^{1-\eta} - 1)/(1-\eta)$, with relative risk aversion $\eta > 0$. Its logarithmic limit occurs at $\eta = 1$. DM effort cost is $c(q,k) = A_c q^{\omega} k^{1-\omega}$, where $A_c > 0$. CM utility and production retain the forms in Section 3, and search cost remains quadratic. We normalize $\bar{A} = 1$ and select the remaining internal parameters jointly.

The external parameters fix the environment in which buyers choose search (Table 1, Panel A). Depreciation is the BEA sample-average ratio of depreciation to net fixed assets. The credit-card share of in-person payment value in the 2018 Diary of Consumer Payment Choice proxies the monitored share. This payment classification does not directly measure monitoring. We set $n = 1/2$, making the search bound one.

Money demand disciplines the scale and curvature of DM demand. We use New-M1 relative to nominal GDP, following Lucas and Nicolini (2015). The opportunity cost is the three-month Treasury-bill rate. An exponential fit to the annual observations supplies the money-demand level and interest elasticity. We evaluate both at the sample mean nominal rate of 5.82 percent. Their model counterpart is

$$\frac{M}{PY} = \frac{(w/\bar{A})q^m c_q(q^m, K)}{Y},$$

Table 1: Annual calibration

Parameter	Role	Value	Source or target
Panel A. Externally set parameters			
β	Discount factor	0.96	Standard annual value
α	Centralized-market capital share	0.33	Standard value
δ	Depreciation rate	0.052	BEA Fixed Assets Tables
ρ	Monitored share	0.25	Diary of Consumer Payment Choice, 2018
n	Buyer share	0.5	Equal numbers of buyers and sellers
$\bar{A}$	Labor disutility	1	Normalization
Panel B. Jointly calibrated parameters			
$\bar{B}$	Centralized-market utility weight	0.397	Hours per household
A_c	Production-cost scale	0.0069	Money-demand level
η	Decentralized-market relative risk aversion	0.148	Money-demand interest elasticity
ω	Production-cost curvature	1.446	Capital–output ratio
$\bar{C}$	Decentralized-market utility weight	0.026	Decentralized-market output share
ν	Search-cost parameter	0.017	Payment-cost proxy

Notes: Internal parameters are rounded. Each is paired with the moment it primarily affects; the moments are matched jointly. The annual calibration sample is 1980–2007.

Table 2: Empirical proxies and calibration targets

Moment	Empirical proxy	Target	Model	Source
Hours per household	0.333	0.333	0.333	Aruoba et al. (2011)
Money–output ratio	0.219	0.219	0.218	New-M1, Lucas and Nicolini (2015)
Money-demand interest elasticity	−0.253	−0.253	−0.253	Fit at the mean nominal rate
Capital–output ratio	3.53	3.50	3.50	Penn World Table 11.0
Decentralized-market output share	0.070	0.10	0.10	BEA retail value added
Search-cost share of output	0.0096	0.005	0.005	Schmiedel et al. (2012)

Notes: Values are rounded. Money-demand moments are evaluated at the mean Treasury-bill rate of 5.82 percent. Output-share and payment-cost observations are proxies for the corresponding model objects.

where M is nominal money, P is the CM price level, and Y is total output valued in CM goods. The production-cost scale primarily affects the money-demand level. DM utility curvature primarily affects its interest elasticity. Figure 3 compares the fitted model schedule with the annual observations.

The remaining targets discipline the scale of capital and decentralized activity. Hours are one third of discretionary time. The capital–output target is 3.50, close to the U.S. sample average of 3.53. We target a DM output share of 0.10, compared with a retail-value-added proxy of 0.070. The search-cost target is 0.005, compared with a retail-payment-cost proxy of 0.0096 in Schmiedel, Kostova, and Ruttenberg (2012). The latter also covers bank, infrastructure, and retailer costs, so it is broader than buyer search. Table 2 reports the distinction between the chosen targets and their empirical proxies.

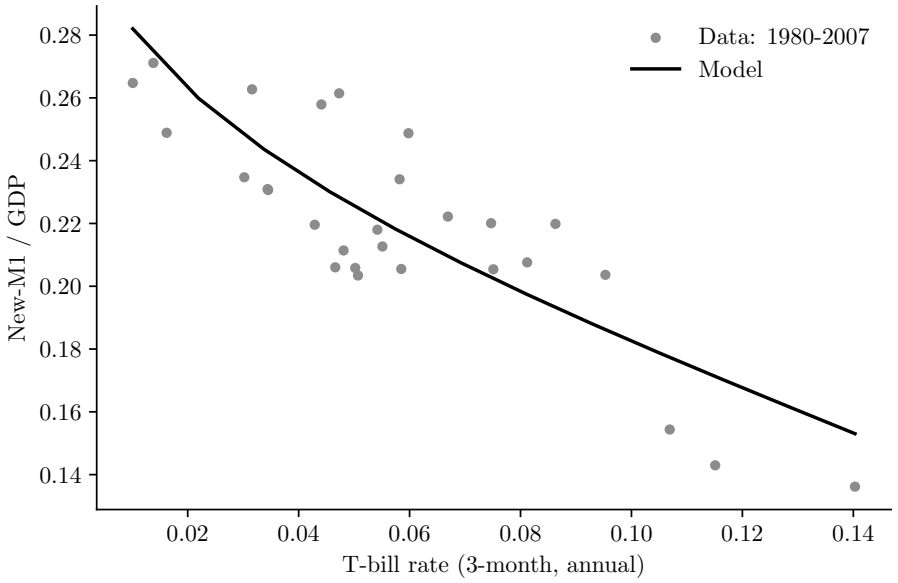


Figure 3: Money demand: data and model. Points show annual New-M1 relative to nominal GDP against the three-month Treasury-bill rate, 1980–2007. The line shows model money demand.

The factor-share identity explains how the output-share target disciplines production-cost curvature. Define the DM output share by

$$s^{DM} = \frac{n\sigma(w/\bar{A})[\rho q^c c_q(q^c, K) + (1-\rho) q^m c_q(q^m, K)]}{Y}.$$

The capital Euler equation implies

$$\frac{K}{Y} = \frac{\alpha(1 - s^{DM}) + \zeta s^{DM}}{1/\beta - 1 + \delta}.$$

The two output targets identify the DM capital share within the joint calibration. The numerator is the output-weighted average of the sectoral capital shares. Given the CM share and the required capital return, matching K/Y and s^{DM} pins down ζ. Production-cost curvature then follows from $\omega = 1/(1-\zeta)$. Separately, the search-cost target values the utility expenditure $n\nu\sigma^2/2$ in CM goods using $w/\bar{A}$. These identities explain which features of the equilibrium each target disciplines.

4.2 Capital and trade along the selected branch

The baseline capital peak marks the point at which extra meetings cease to compensate for smaller monetary purchases. At low nominal rates in Figure 4, buyers search harder and sellers use capital more often. The increased frequency of production initially supports more capital. As the nominal rate rises, monetary purchases continue to shrink. The associated loss in effort savings eventually outweighs the frequency gain, producing a single peak on the computed path.

The later search peak reflects the difference between the buyer's reward from a meeting and the seller's return to capital. Buyers can still gain from the price saving when the extra meetings no longer compensate sellers for the smaller production saving per meeting. Search therefore continues rising after

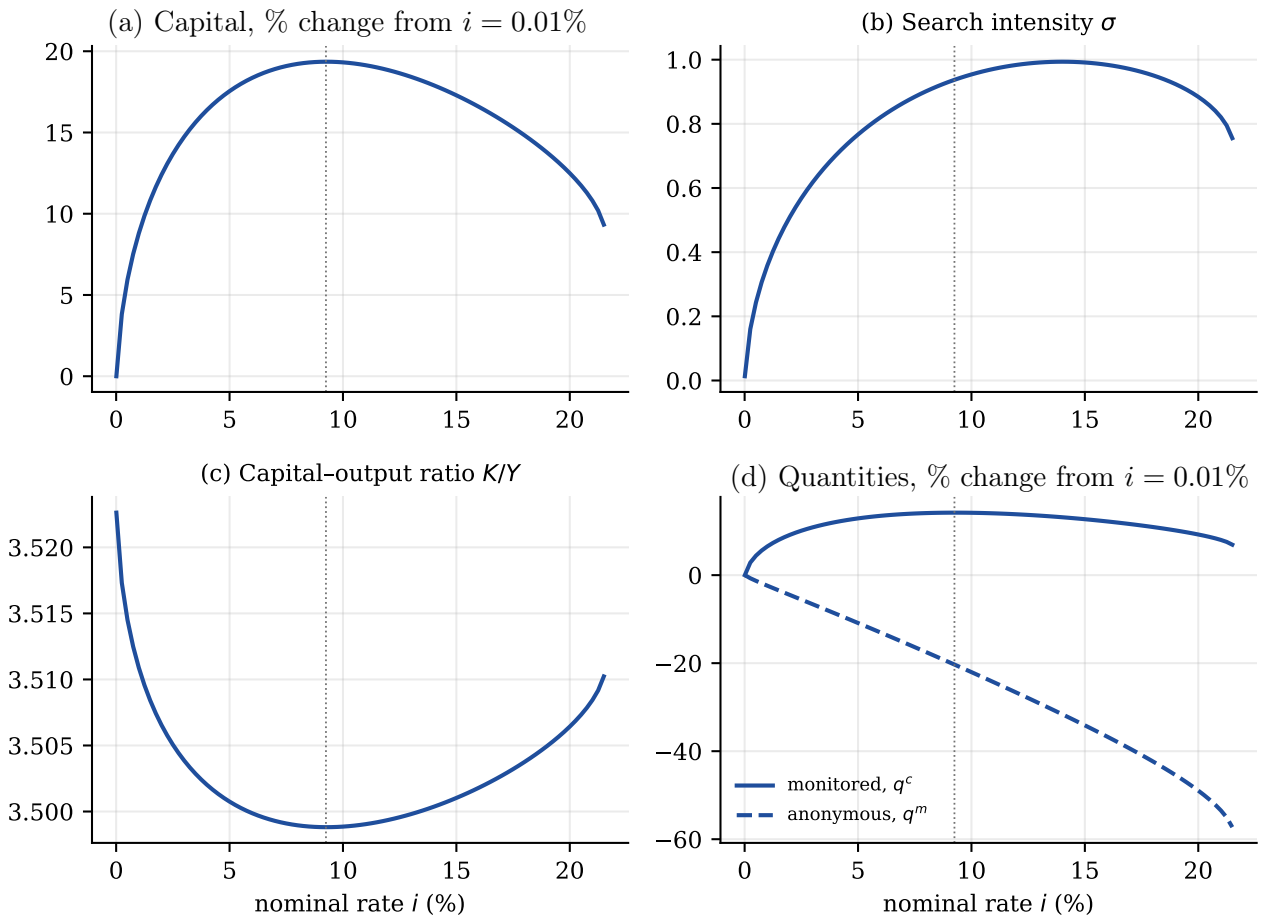


Figure 4: Baseline capital and trade. Baseline calibration, highest-search branch. (a) Capital relative to the low-rate endpoint; (b) search intensity; (c) capital–output ratio; (d) credit and monetary quantities relative to the low-rate endpoint. The vertical line marks the capital peak. Percentage changes use $i = 0.0001$ (0.01 percent) as the reference.

capital starts to fall on the computed path. Credit quantities follow capital because capital lowers the production cost of a credit trade. The calibrated share restriction makes the K/Y minimum coincide with the capital peak. The sectoral capital shares are close, so the ratio moves much less than the capital stock.

Lower trading frictions raise the level of capital while removing its positive response to inflation in this experiment. Reducing the search cost to one fifth puts buyers at the meeting bound over the reported range. Their higher trading frequency supports more capital than the baseline at each nominal rate. Further increases in the rate cannot create more meetings, so smaller monetary purchases reduce capital. Credit purchases then fall with capital, and the factor-share restriction makes K/Y rise. Figure 9 in Appendix B reports the experiment, which holds all other parameters fixed.

The monitored-share experiment changes how often money is used, which also affects the search incentive. At given search, a higher ρ means fewer monetary meetings. The holding cost per monetary use, $i/[n\sigma(1-\rho)]$, consequently rises. In the logarithmic analytical case, this shortens the range over which the direct price incentive is positive. In the CRRA numerical exercise, a higher ρ moves the

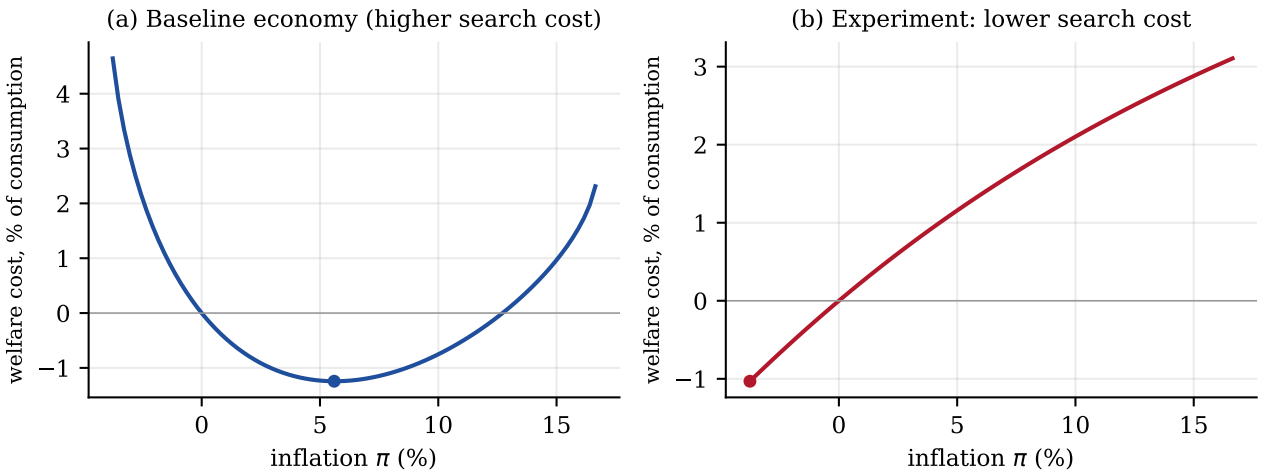


Figure 5: Stationary welfare and trading frictions. Consumption-equivalent cost relative to zero inflation. (a) Baseline calibration; (b) search cost reduced to one fifth, with other parameters fixed. Negative values indicate welfare gains. Dots mark the best allocations on the displayed numerical grids, which begin at $i = 0.0025$ (0.25 percent), above the Friedman rule.

capital peak toward the Friedman rule. It also reduces the range of monetary equilibria found. Because search and capital both adjust, these computed patterns do not establish a general comparative-static result for ρ.

4.3 Stationary welfare and the trading friction

Welfare depends on the total gains from the allocation, rather than capital accumulation alone. Let inflation be $\pi = \tau$, so the nominal rate is $i = (1+\pi)/\beta - 1$. Stationary welfare is

$$\mathcal{W}(\pi) = \frac{1}{1-\beta}\Big\{\bar{B}\ln x - \bar{A}H + n\sigma[\rho(u(q^c) - c(q^c,K)) + (1-\rho)(u(q^m) - c(q^m,K))] - n\mu(\sigma)\Big\}. \quad (4.1)$$

The DM contribution is the expected utility from trade net of production effort and search expenditure. Payments cancel because a buyer's expense is a seller's receipt. A lower price can therefore encourage private search without directly increasing total surplus. More meetings can still improve welfare by generating additional gains from trade. Those gains must be weighed against the effort spent searching, including by buyers who find no seller.

The consumption-equivalent measure expresses a welfare difference as a proportional change in consumption at zero inflation. Following Lucas (2000) and Lagos and Wright (2005), we multiply reference CM consumption and both DM quantities by $\Delta > 0$. Labor, effort, search, and capital stay at their reference values. The scale is chosen so that $\mathcal{W}(\pi) = \mathcal{W}_\Delta(0)$, and the reported welfare cost is $1 - \Delta$. A negative value means the alternative allocation yields higher welfare than zero inflation. Appendix A.7.2 gives the formula. Zero inflation is the reference allocation; the Friedman rule instead has $\pi = \beta - 1$.

Welfare continues improving after capital begins to fall on the computed baseline branch. At the lower rates in Figure 5(a), the gains from additional trade exceed the losses from smaller purchases and

the additional search expenditure. Search keeps rising beyond the capital peak, so trading opportunities can continue expanding even with less capital. The distinct peaks show why the capital response alone cannot rank allocations by welfare. At higher rates, the contraction of monetary trade dominates the remaining frequency gain and welfare declines.

The economy with lower trading frictions has no additional meetings to offset inflation's effect on purchases. Search is already at its bound along the computed path. Monetary quantities shrink as the nominal rate rises, and the accompanying fall in capital also reduces credit quantities. The welfare calculation ranks these changes negatively, giving the Friedman rule the highest computed stationary welfare. The $i = 0$ allocation is evaluated separately because the plotted grid starts at $i = 0.0025$.

The baseline welfare peak ranks allocations only within the computed positive-search branch. The solver finds no positive-search Friedman-rule allocation at $i = 0$. A dense grid over admissible search also yields a positive reduced search residual throughout. These numerical checks leave the benchmark unresolved; they do not prove nonexistence. A zero-search or nonmonetary allocation could be the relevant alternative, but must first be characterized.

Trade participation is a second unresolved requirement for the low-rate baseline allocations. Under the stated utility normalization, conditional credit-trade surplus is negative near the baseline endpoint. A buyer who can reject that meeting and receive the normalized no-trade payoff would decline it. Allowing rejection changes the surplus that motivates search, so the equilibrium conditions and calibration must be reconsidered together. Until participation is specified, the reported paths are solutions to the stated equations rather than a complete equilibrium characterization.

The private search condition does not by itself establish the socially preferred meeting frequency. Buyers count their own surplus, while an additional meeting also benefits the seller. A planner would value both gains and account for the resource cost of generating the meeting. This difference motivates a possible search wedge, but its equilibrium size and policy implications require a planner solution. Transitions and changes in equilibrium selection would also enter a full policy evaluation.

5 Suggestive evidence

The empirical exercise asks whether capital–output patterns resemble the model's predictions across different trading environments. Under the factor-share restriction, a capital peak produces a minimum in K/Y. When capital instead declines with inflation, the ratio rises. Both shapes occur in U.S. and international data. Their resemblance to the model motivates a descriptive comparison; identifying the mechanism requires independent variation in trading costs.

5.1 Data and construction

The empirical comparison pairs capital–output ratios with consumer-price inflation. The U.S. ratio uses the BEA net stock of fixed assets at current cost relative to nominal GDP. For international observations, the Global Macro Database supplies perpetual-inventory capital (Müller, Xu, Lehbib, and Chen, 2025). The Penn World Table supplies capital at constant national prices (Feenstra, Inklaar, and Timmer, 2015). Additional long-run series come from the Jordà–Schularick–Taylor database (Jordà, Schularick, and Taylor, 2017). Table 3 in Appendix C documents coverage and construction.

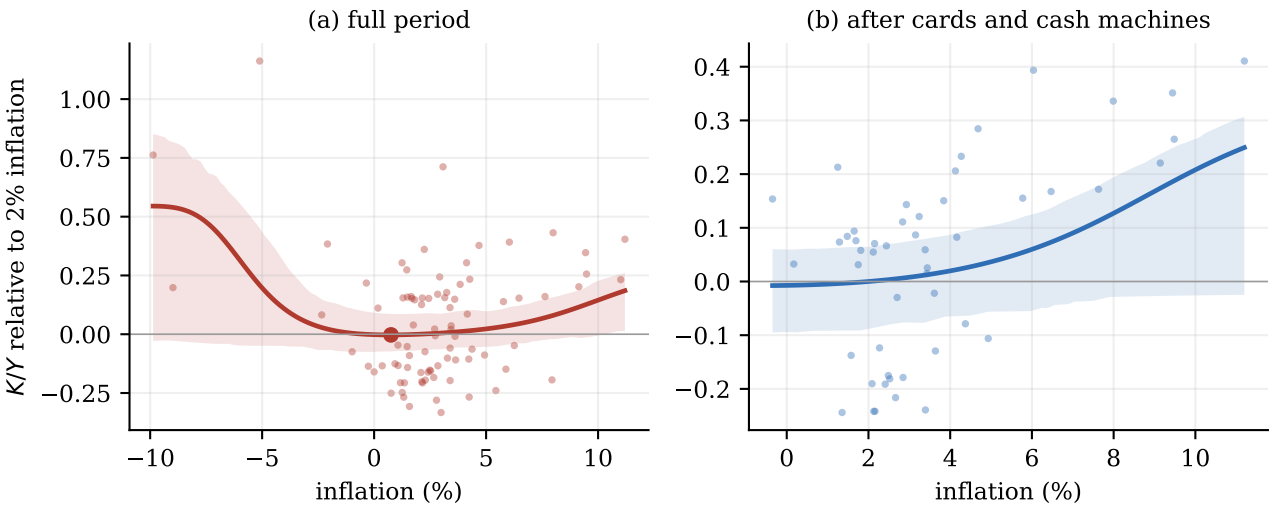


Figure 6: U.S. capital–output schedules. (a) 1929–2024, excluding 1939–1948; (b) 1975–2024. BEA net fixed assets at current cost relative to nominal GDP, with a separate linear trend removed in each sample. Points are annual observations; lines are Gaussian kernel means with bandwidth 3 percentage points; bands are 90 percent pointwise intervals from a five-year block bootstrap. Values are relative to the fitted ratio at 2 percent inflation.

Payment access provides a proxy for the ease of trade rather than a direct measure of the model's search cost. We use cash-machine density, bank-branch density, account ownership, and digital-payment use (World Bank, 2025a,b). These services can reduce the cost of obtaining or using means of payment, as motivated by Alvarez and Lippi (2009). The model abstracts from withdrawal trips and payment-instrument choice, so the mapping to ν is indirect. The proxies' recent coverage also limits comparisons with historical schedules.

5.2 U.S. and international schedules

The U.S. schedule changes shape when the sample is restricted to recent decades. The long sample has an interior minimum. The 1975–2024 schedule instead rises over the observed range. Figure 6 plots kernel-weighted means after removing a separate linear trend from K/Y in each sample. The output collapse during the Great Depression influences the long-run left arm. Inflation support also differs across the samples. These differences prevent attributing the change in shape to payment innovation alone.

The pooled international schedule has a shallow, imprecisely estimated interior minimum. We use non-overlapping three-year averages to reduce annual fluctuations. Before pooling, we remove country-specific linear trends from both inflation and K/Y. Figure 7(a) reports the Penn World Table schedule. Annual observations and five-year averages yield similarly shallow shapes. The small movement in K/Y resembles the calibration with close sectoral capital shares. Common shocks and other determinants of capital remain possible explanations after detrending.

Grouping countries by schedule shape describes heterogeneity but cannot independently test the model. A U-shaped classification requires an interior minimum below both endpoints. Among the remaining countries, the rising classification requires the final value to exceed the initial value. It does

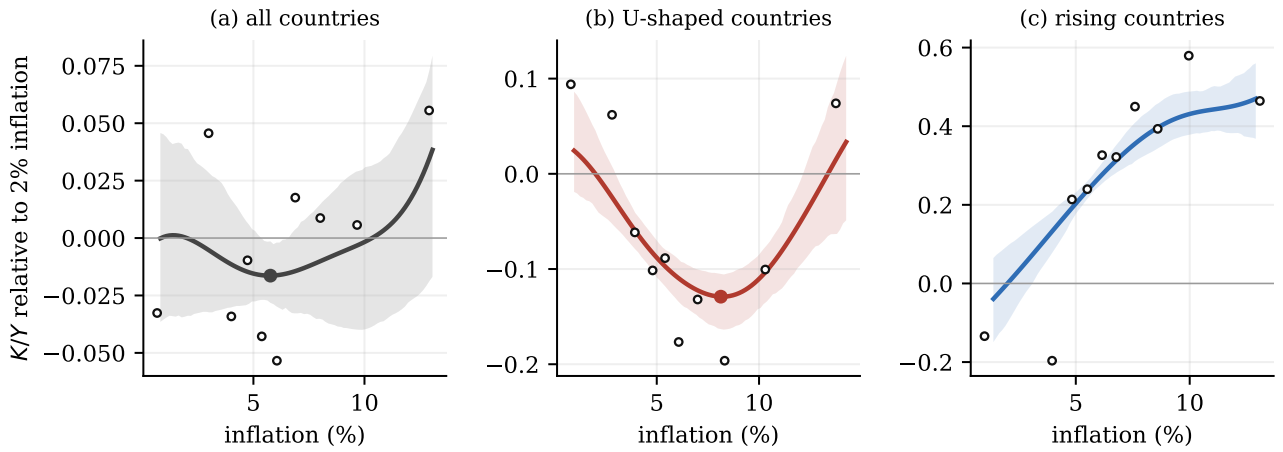


Figure 7: International capital–output schedules. Penn World Table capital relative to output at constant national prices, 1970–2023; inflation restricted to −5 to 20 percent; three-year averages with country-specific linear trends removed from both variables. (a) All eligible countries; (b) U-shaped classifications in both datasets; (c) rising classifications in both. Lines are Gaussian kernel means with bandwidth 2 percentage points; bands are 90 percent pointwise country-bootstrap intervals; open points are decile means. Values are relative to the fitted ratio at 2 percent inflation. Filled points mark interior minima.

not impose monotonicity between those endpoints. We retain classifications that agree across the Global Macro Database and Penn World Table. Panels (b) and (c) plot the corresponding pooled schedules, whose contrast partly follows from this outcome-based selection.

The selected country examples illustrate how payment access might relate to the availability of the frequency response. Figure 10 in Appendix C pairs low-access countries with U-shaped schedules and high-access countries with rising schedules. Cash-machine density and account ownership distinguish the access groups. Both access and schedule shape enter the selection, so these examples do not establish an unconditional relation.

The descriptive patterns leave the trading-cost explanation open to competing interpretations. Reshuffling inflation within countries produces at least as many U-shaped schedules as the data. Historical output shocks influence the U.S. pattern. Payment access also correlates with income and institutions. A test of the mechanism would need variation in trading costs that can be separated from these other influences.

6 Conclusion

Inflation can raise capital when its effect on trading frequency outweighs its effect on the size of each trade. Competitive price reductions can increase buyer surplus even as monetary purchases shrink. Buyers then search harder, giving sellers more opportunities to save effort with capital. The analytical surplus threshold identifies when this frequency gain dominates near the Friedman rule on an existing regular interior highest-search branch. Capital reaches an interior maximum if the balance later reverses along that branch. When search remains at its upper bound, no additional meetings are available and capital declines.

Lower trading frictions can support more capital while eliminating its positive response to inflation. The calibration illustrates this distinction by reducing the search cost to one fifth of its baseline value. Search reaches its bound over the reported range, and the Friedman rule becomes the best computed stationary allocation. The baseline instead has an interior welfare peak within its computed branch. Its comparison remains conditional on an unverified Friedman-rule allocation and unresolved low-rate participation. The descriptive evidence motivates further study without establishing the search channel.

A full policy assessment requires the private search incentive to be compared with the total gain from a meeting. A planner solution would establish that comparison. Completing the baseline allocation set also requires participation and zero-search analysis. Transition analysis would measure the cost of moving between them. Empirically, direct measures of trading costs could test whether payment improvements change the frequency of capital use. The factor-share restriction must also be assessed when translating a capital response into a prediction for K/Y.

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A Proofs

A.1 Proof of Proposition 1

Step 1: the capital curve. The capital–labor ratio lies in $(0, \hat{k}_{\max})$, where $\mathcal{H}(\hat{k}_{\max}) = 0$, that is, $\hat{k}_{\max} = \delta^{1/(\alpha-1)}$. By the proof of Lemma 1, $J_{\hat{k}} > 0$ at every $(\hat{k}, \sigma, i)$. As $\hat{k} \to 0$, the rental rate $\alpha\hat{k}^{\alpha-1}$ grows without bound and $J \to -\infty$; as $\hat{k} \to \hat{k}_{\max}$, $\mathcal{H} \to 0$ and $J \to 1/\beta - 1 + \delta(1-\alpha) > 0$. Hence for every $\sigma \in [0, \bar{\sigma}]$ and $i \geq 0$ the equation $J = 0$ has a unique solution $\kappa(\sigma, i) \in (0, \hat{k}_{\max})$, and κ is continuous by the implicit function theorem.

Step 2: the reduced search condition. $\Phi(\sigma) = \mathcal{T}(\kappa(\sigma, i), \sigma; i)$ is continuous on $(0, \bar{\sigma}]$. As $\sigma \to 0$ with $i > 0$, $\gamma \to 0$, so $\ln q^m = \ln q^c + (\ln \gamma)/\omega \to -\infty$ while $\ln q^c(\kappa(\sigma, i))$ stays bounded; since $\rho < 1$, the anonymous surplus in Equation (3.1) tends to $-\infty$ and $\Phi(\sigma) \to +\infty$.

Step 3: two equilibria. Let $\Phi(\sigma_1) < 0$. By Step 2 there is $\sigma_0 \in (0, \sigma_1)$ with $\Phi(\sigma_0) > 0$, so by the intermediate value theorem Φ has a zero in (σ_0, σ_1). On $[\sigma_1, \bar{\sigma}]$, either $\Phi(\bar{\sigma}) > 0$, and Φ has a zero in $(\sigma_1, \bar{\sigma})$, or $\Phi(\bar{\sigma}) \leq 0$, and at $\sigma = \bar{\sigma}$ the expected surplus is at least the marginal cost of search, so the upper bound solves the buyer's problem. At each such σ, $\hat{k} = \kappa(\sigma, i)$ satisfies the capital Euler equation, and the CM conditions deliver the rest of Definition 1.

Step 4: the highest equilibrium. Let $\Phi(\bar{\sigma}) > 0$ and let σ^* be the largest zero of Φ. Then $\Phi > 0$ on $(\sigma^*, \bar{\sigma}]$, so $\Phi'(\sigma^*) \geq 0$. Differentiating, $\Phi' = T_\sigma + T_{\hat{k}}\kappa_\sigma = T_\sigma - T_{\hat{k}}J_\sigma/J_{\hat{k}} = -D/J_{\hat{k}}$, where $D = T_{\hat{k}}J_\sigma - T_\sigma J_{\hat{k}}$ is the determinant of Lemma 3, and $J_{\hat{k}} > 0$ gives $D \leq 0$. The argument holds at every $i \geq 0$. □

A.2 Proof of Lemma 1

Derivatives of γ. From $\gamma = n\sigma(1-\rho)/[n\sigma(1-\rho) + i]$, $\gamma_i = -\gamma^2/(n\sigma(1-\rho))$ and $\gamma_\sigma = \gamma(1-\gamma)/\sigma$. At $i = 0$, $\gamma = 1$, so $\gamma_i = -1/(n\sigma(1-\rho))$, $\gamma_\sigma = 0$, and $q^m = q^c\gamma^{1/\omega} = q^c$.

Search curve. Since $\ln q^m = \ln q^c + (\ln \gamma)/\omega$, the search curve (3.1) depends on (σ, i) through $\nu\sigma$ and γ, with $T_\gamma = \partial\mathcal{T}/\partial\gamma = -\bar{C}(1-\rho)[1/(\omega\gamma) - 1]$, which equals $\zeta\bar{C}(1-\rho)$ at $\gamma = 1$. Hence $T_i = \zeta\bar{C}(1-\rho)\gamma_i = -\zeta\bar{C}/(n\sigma)$ and $T_\sigma = \nu + \zeta\bar{C}(1-\rho)\gamma_\sigma = \nu$.

Capital curve. From Equation (3.2), $J_i = -n\sigma(1-\rho)\gamma_i\,(\zeta\bar{C}/\bar{B})\mathcal{H}(\hat{k})$, which equals $(\zeta\bar{C}/\bar{B})\mathcal{H}(\hat{k}) > 0$ at $i = 0$ because $\mathcal{H}(\hat{k}) > 0$. Differentiating in $\hat{k}$,

$$J_{\hat{k}} = (1-\alpha)\hat{k}^{\alpha-2}\Big[\alpha + n\sigma\mathcal{P}(\sigma, i)\frac{\zeta\bar{C}}{\bar{B}}\Big] > 0$$

at every $i \geq 0$, and $J_\sigma = -n(\zeta\bar{C}/\bar{B})\mathcal{H}(\hat{k})[\mathcal{P} + \sigma(1-\rho)\gamma_\sigma] < 0$, which equals $-n(\zeta\bar{C}/\bar{B})\mathcal{H}(\hat{k})$ at $i = 0$.

Curve shifts. Holding $\hat{k}$ fixed, $\mathcal{T} = 0$ gives $d\sigma/di = -T_i/T_\sigma = \zeta\bar{C}/(n\sigma\nu) > 0$; holding σ fixed, $J = 0$ gives $d\hat{k}/di = -J_i/J_{\hat{k}} < 0$. □

A.3 The hot-potato condition away from the Friedman rule

Since $\ln q^m = \ln q^c + (\ln \gamma)/\omega$, the search curve (3.1) depends on i only through γ, with $T_\gamma = -\bar{C}(1-\rho)[1/(\omega\gamma) - 1]$. Because $\gamma_i < 0$, $T_i = T_\gamma\gamma_i < 0$ exactly when $T_\gamma > 0$, that is, when $\gamma > 1/\omega$. Since $1/\gamma = 1 + i/(n\sigma(1-\rho))$, this is $i/(n\sigma(1-\rho)) < \omega - 1$, and $\omega - 1$ is the elasticity of marginal cost of the power cost function. With a general cost function, differentiating $S^m = u(q^m) - q^m c_q(q^m, K)$ and

using Equation (2.13) gives

$$\frac{\partial S^m}{\partial q^m} = c_q(q^m, K)\Big[\frac{i}{n\sigma(1-\rho)} - \frac{q^m c_{qq}(q^m, K)}{c_q(q^m, K)}\Big],$$

and since q^m falls with i, the anonymous surplus rises with i, and $T_i < 0$, exactly when $i/(n\sigma(1-\rho)) < q^m c_{qq}(q^m, K)/c_q(q^m, K)$. □

A.4 Proof of Lemma 2

Preliminaries. The buyer's problem $\max_{\sigma\in[0,\bar\sigma]}\{-\nu\sigma^2/2 + \sigma E\}$, with E the expected surplus, has the solution $\sigma = \min\{\max\{E/\nu, 0\}, \bar\sigma\}$; search is at its upper bound exactly when $E \geq \nu\bar\sigma$ with capital on the capital curve. Let $\Phi_i(\sigma) = \mathcal{T}(\kappa(\sigma, i), \sigma; i)$. The highest-search equilibrium is the upper bound $\bar\sigma$ if $\Phi_i(\bar\sigma) \leq 0$ and the largest zero of Φ_i otherwise. Let $\ell(\hat k) = \partial \ln q^c(\hat k)/\partial \hat k > 0$; more capital lowers the seller's marginal cost and enlarges every trade.

Fact 1: the surplus at the Friedman rule rises with search. At $i = 0$, $\gamma = 1$ and $q^m = q^c$, so the expected surplus is $\bar C[\ln q^c(\hat k) - 1]$ and $s(\sigma) = \bar C[\ln q^c(\kappa(\sigma, 0)) - 1]$. Since $\kappa_\sigma = -J_\sigma/J_{\hat k} > 0$ by the proof of Lemma 1, $s'(\sigma) = \bar C\ell(\kappa)\kappa_\sigma > 0$.

Fact 2: the upper bound. $\Phi_0(\bar\sigma) = \nu\bar\sigma - s(\bar\sigma)$, so search is at its upper bound at $i = 0$ exactly when $\nu \leq \nu_c = s(\bar\sigma)/\bar\sigma$.

Fact 3: search falls with its cost. For $\nu > \nu_c$, let $\sigma^{FR}(\nu)$ be the largest zero of $\Phi_0(\sigma) = \nu\sigma - s(\sigma)$. Then $\Phi_0 > 0$ on $(\sigma^{FR}(\nu), \bar\sigma]$. For $\nu' > \nu$, $\nu'\sigma - s(\sigma) > \nu\sigma - s(\sigma) \geq 0$ on $[\sigma^{FR}(\nu), \bar\sigma]$, so the largest zero for ν', if any, lies below $\sigma^{FR}(\nu)$: σ^{FR} is strictly decreasing in ν.

Part (a). If $\nu < \nu_c$, $\Phi_0(\bar\sigma) < 0$ by Fact 2. Since κ and γ are continuous in i at $\sigma = \bar\sigma > 0$, $\Phi_i(\bar\sigma) < 0$ for small i, so search is at its upper bound, which is the highest possible search. With $\sigma = \bar\sigma$ fixed, $\hat k(i) = \kappa(\bar\sigma, i)$ and $d\hat k/di = -J_i/J_{\hat k}$, where $J_i = -n\bar\sigma(1-\rho)\gamma_i(\zeta\bar C/\bar B)\mathcal{H} > 0$ because $\gamma_i < 0$ and $\mathcal{H} > 0$. Hence $d\hat k/di < 0$.

Part (b). If $\nu > \nu_c$, search is interior and, by Step 4 of Appendix A.1, $D \leq 0$; regularity gives $D < 0$. By the implicit function theorem $d\hat k/di = N/D$ with $N = T_\sigma J_i - J_\sigma T_i$. At $i = 0$ the proof of Lemma 1 gives $T_i = -\zeta\bar C/(n\sigma)$, $T_\sigma = \nu$, $J_i = (\zeta\bar C/\bar B)\mathcal{H}$ and $J_\sigma = -n(\zeta\bar C/\bar B)\mathcal{H}$, so

$$N = \nu\frac{\zeta\bar C}{\bar B}\mathcal{H} - n\frac{\zeta\bar C}{\bar B}\mathcal{H}\cdot\frac{\zeta\bar C}{n\sigma} = \frac{\zeta\bar C}{\bar B}\mathcal{H}\Big(\nu - \frac{\zeta\bar C}{\sigma^{FR}}\Big).$$

Since $D < 0$, $d\hat k/di > 0$ exactly when $N < 0$, that is, when $\nu\sigma^{FR} < \zeta\bar C$. At $i = 0$ the search condition reads $\nu\sigma^{FR} = s(\sigma^{FR})$, which gives the stated condition.

Part (c). Facts 1 and 3 imply that $s(\sigma^{FR}(\nu))$ strictly decreases among costs for which the selected positive interior equilibrium exists. This ordering does not establish existence at every cost. On a connected interval with a continuous selected branch, a crossing of $s(\sigma^{FR}) = \zeta\bar C$ is unique by strict monotonicity. If it occurs at $\hat\nu$, part (b) gives a negative local capital response below it and a positive response above it. At an interior regular crossing, the derivative is zero. If $s(\bar\sigma) \leq \zeta\bar C$, then strict increase of s and $\sigma^{FR} < \bar\sigma$ give $s(\sigma^{FR}) < \zeta\bar C$ at every existing interior equilibrium. If $s(0) \geq \zeta\bar C$, positivity of search gives $s(\sigma^{FR}) > \zeta\bar C$. If $s(0) < \zeta\bar C < s(\bar\sigma)$, strict monotonicity and continuity of s

imply a unique $\sigma^* \in (0, \bar{\sigma})$ satisfying $s(\sigma^*) = \zeta \bar{C}$. A selected-branch crossing, if it exists, then satisfies $\hat{\nu} = \zeta \bar{C}/\sigma^*$. This expression is conditional on that search level belonging to the selected branch.

Part (d). Let $R = 1/\beta - 1 + \delta$ and $Q(\hat{k}) = R - (\alpha - \zeta)\hat{k}^{\alpha-1}$, which is positive at an equilibrium with $Y > 0$. Differentiate (3.3):

$$\begin{aligned} \frac{d(K/Y)}{d\hat{k}} &= -\frac{\zeta}{Q(\hat{k})^2} Q'(\hat{k}), \\ Q'(\hat{k}) &= (\alpha - \zeta)(1 - \alpha)\hat{k}^{\alpha-2}, \\ \frac{d(K/Y)}{d\hat{k}} &= -\frac{\zeta(\alpha - \zeta)(1 - \alpha)\hat{k}^{\alpha-2}}{Q(\hat{k})^2}. \end{aligned}$$

The derivative is negative for $\zeta < \alpha$, zero for equal shares, and positive for $\zeta > \alpha$. □

A.5 Proof of Lemma 3

The ratios. By the implicit function theorem applied to $\mathcal{T} = 0$ and $J = 0$ along a regular branch, $d\hat{k}/di = N/D$ and $d\sigma/di = M/D$ with

$$N = T_\sigma J_i - J_\sigma T_i, \qquad M = J_{\hat{k}} T_i - T_{\hat{k}} J_i, \qquad D = T_{\hat{k}} J_\sigma - T_\sigma J_{\hat{k}}.$$

Let $A = \zeta \bar{C}/\bar{B}$ and $T_\gamma = -\bar{C}(1 - \rho)[1/(\omega\gamma) - 1]$ (Appendix A.3). Both surpluses in Equation (3.1) depend on $\hat{k}$ only through $\ln q^c(\hat{k})$, so $T_{\hat{k}} = -\bar{C}\ell$; also $T_i = T_\gamma \gamma_i$ and $T_\sigma = \nu + T_\gamma \gamma_\sigma$, while the capital curve gives $J_i = -n\sigma(1 - \rho)\gamma_i A\mathcal{H}$ and $J_\sigma = -nA\mathcal{H}[\mathcal{P} + \sigma(1 - \rho)\gamma_\sigma]$. Then

$$N = nA\mathcal{H}\gamma_i\big\{-\sigma(1 - \rho)(\nu + T_\gamma\gamma_\sigma) + [\mathcal{P} + \sigma(1 - \rho)\gamma_\sigma]T_\gamma\big\} = nA\mathcal{H}\gamma_i\big[-\sigma(1 - \rho)\nu + \mathcal{P}T_\gamma\big],$$

because the two terms in γ_σ cancel, and substituting T_γ gives $N = nA\mathcal{H}[-(1-\rho)\gamma_i][\sigma\nu + \bar{C}\mathcal{P}(1/(\omega\gamma) - 1)]$, the stated form. Similarly,

$$D = n\,\ell\,\frac{\zeta\bar{C}^2}{\bar{B}}\mathcal{H}\big[\mathcal{P} + \sigma(1 - \rho)\gamma_\sigma\big] - J_{\hat{k}}\Big[\nu - \bar{C}(1 - \rho)\Big(\frac{1}{\omega\gamma} - 1\Big)\gamma_\sigma\Big],$$

$$M = J_{\hat{k}} T_\gamma \gamma_i - \bar{C}\ell\, n\sigma(1 - \rho)\gamma_i A\mathcal{H} = \big[-(1 - \rho)\gamma_i\big]\,\bar{C}\Big[J_{\hat{k}}\Big(\frac{1}{\omega\gamma} - 1\Big) + n\sigma\ell A\mathcal{H}\Big].$$

The sign of D. Along a branch of highest-search equilibria with interior search, $\Phi_i(\bar{\sigma}) > 0$ and $\sigma(i)$ is the largest zero of Φ_i at every i, so Step 4 of Appendix A.1 gives $D(i) \le 0$, and $D(i) < 0$ where the branch is regular.

The sign of N. Since $-(1 - \rho)\gamma_i > 0$ and $\mathcal{H} > 0$, N has the sign of $\mathcal{N}$, and with $D < 0$ capital rises exactly when $\mathcal{N} < 0$. Write the frequency term as $f(\gamma) = \bar{C}[\rho + (1 - \rho)\gamma][1/(\omega\gamma) - 1]$. Then

$$f'(\gamma) = \bar{C}\Big[(1 - \rho)\Big(\frac{1}{\omega\gamma} - 1\Big) - \frac{\rho + (1 - \rho)\gamma}{\omega\gamma^2}\Big] = -\bar{C}\Big[(1 - \rho) + \frac{\rho}{\omega\gamma^2}\Big] < 0,$$

so f is strictly decreasing, $f(1) = -\zeta\bar{C}$, and $f(\gamma) \ge 0$ exactly when $\gamma \le 1/\omega$, that is, when $i \ge (\omega - 1)n\sigma(1 - \rho)$. At such rates $\mathcal{N} = \sigma\nu + f(\gamma) > 0$.

The level of search and capital. Fix i and write $\Phi_i(\sigma;\nu) = \nu\sigma - E(\kappa(\sigma,i),\sigma;i)$, so that $\Phi_i(\sigma;\nu') = \Phi_i(\sigma;\nu) - (\nu-\nu')\sigma < \Phi_i(\sigma;\nu)$ for $\nu' < \nu$ and $\sigma > 0$. If search is at its upper bound under ν, $\Phi_i(\bar\sigma;\nu) \le 0$, then $\Phi_i(\bar\sigma;\nu') < 0$ and search is at its upper bound under ν' as well. If search is interior under ν, at the largest zero $\sigma(\nu)$, then $\Phi_i(\sigma(\nu);\nu') < 0$, so under ν' either $\Phi_i(\cdot;\nu')$ has a zero above $\sigma(\nu)$ or it is negative at $\bar\sigma$; in both cases the highest-search equilibrium has $\sigma(\nu') > \sigma(\nu)$. Capital is $\hat k = \kappa(\sigma,i)$, and $\kappa_\sigma = -J_\sigma/J_{\hat k} > 0$, so capital rises with search; K/Y falls with $\hat k$ by part (d) of Lemma 2. No regularity is needed for this argument. □

A.6 Proof of Proposition 2

Part (a). The surplus condition and Lemma 2(b) give $\hat k'(0) > 0$. At i_h, the direct hot-potato condition fails, so Lemma 3 gives $\mathcal{N}(i_h) > 0$. Regularity on the selected branch gives $D(i_h) < 0$, hence $\hat k'(i_h) < 0$. Continuity implies that $\hat k$ attains a maximum on $[0, i_h]$. A positive right derivative at zero rules out the left endpoint; a negative derivative at i_h rules out the right endpoint. Thus a maximum lies in $(0, i_h)$. If N changes sign exactly once on I, $D < 0$ makes the derivative change from positive to negative exactly once, yielding a unique maximum.

Part (b). With $\sigma = \bar\sigma$, capital satisfies $\hat k(i) = \kappa(\bar\sigma, i)$. Differentiating $J = 0$ gives

$$\hat k'(i) = -\frac{J_i}{J_{\hat k}}, \qquad J_i = -n\bar\sigma(1-\rho)\gamma_i \frac{\zeta \bar C}{\bar B}\mathcal{H} > 0.$$

Since $J_{\hat k} > 0$, the derivative is negative. The upper bound is optimal throughout I exactly when $\nu \le \inf_{i\in I} E(\kappa(\bar\sigma,i),\bar\sigma;i)/\bar\sigma$. This argument uses the logarithmic analytical specification; the quantitative specification is evaluated numerically.

Part (c). Under $\zeta < \alpha$, Lemma 2(d) gives a strictly decreasing mapping from $\hat k$ to K/Y. Maxima of the former are minima of the latter, and a strictly decreasing capital-intensity path implies a strictly increasing ratio. □

A.7 Extra details

A.7.1 The CM block and the capital–output ratio

Eliminating labor from (2.2) makes the linearity used in the main text explicit. Rearrange the budget constraint for hours and substitute into utility:

$$\begin{aligned} wh &= x + (1+\tau)m' + k' - m + b - (1+r-\delta)k - T, \\ W(m,b,k) &= \frac{\bar A}{w}[m - b + (1+r-\delta)k + T] \\ &\quad + \max_{x,m',k'\ge 0}\Big\{U(x) - \frac{\bar A}{w}[x + (1+\tau)m' + k'] + \beta V(m',k')\Big\}. \end{aligned}$$

Differentiating with respect to the choices gives (2.3); differentiating incoming wealth gives (2.4). These steps require the maintained interior labor solution.

The logarithmic CM specification delivers the capital mapping used in the analytical section. From $U'(x) = \bar{A}/w$, $w = (1-\alpha)\hat{k}^{\alpha}$, and $x = F - \delta K$,

$$\begin{aligned} x &= \frac{\bar{B}}{\bar{A}}(1-\alpha)\hat{k}^{\alpha}, \\ x &= K(\hat{k}^{\alpha-1} - \delta), \\ K(\hat{k}) &= \frac{\bar{B}(1-\alpha)\hat{k}^{\alpha}}{\bar{A}(\hat{k}^{\alpha-1} - \delta)} = \frac{\bar{B}(1-\alpha)\hat{k}}{\bar{A}(1 - \delta\hat{k}^{1-\alpha})}. \end{aligned}$$

The admissible domain is $0 < \hat{k} < \delta^{1/(\alpha-1)}$, where the denominator is positive. Writing $a_0 = \bar{B}(1-\alpha)/\bar{A}$,

$$K'(\hat{k}) = \frac{a_0[1 - \alpha\delta\hat{k}^{1-\alpha}]}{(1 - \delta\hat{k}^{1-\alpha})^2} > 0, \qquad \ell(\hat{k}) = \frac{\zeta K'(\hat{k})}{K(\hat{k})} > 0.$$

Thus the capital level and capital intensity move in the same direction under this CM specification.

The capital–output identity uses the power cost function and does not require logarithmic DM utility. Differentiate $c(q, K) = A_c q^{\omega} K^{1-\omega}$ and use $\zeta = (\omega - 1)/\omega$:

$$\begin{aligned} -c_k(q, K) &= (\omega - 1)A_c q^{\omega} K^{-\omega}, \\ \frac{q c_q(q, K)}{K} &= \omega A_c q^{\omega} K^{-\omega}, \\ -c_k(q, K) &= \zeta \frac{q c_q(q, K)}{K}. \end{aligned}$$

Define output in the DM at transaction prices by

$$Y^{DM} = n\sigma \frac{w}{\bar{A}}[\rho q^c c_q(q^c, K) + (1-\rho) q^m c_q(q^m, K)], \qquad Y = F + Y^{DM}.$$

Substitute the cost derivative and $r = \alpha F/K$ into (2.14), then collect terms:

$$\begin{aligned} \frac{1}{\beta} - 1 + \delta &= \alpha \frac{F}{K} + \zeta \frac{Y^{DM}}{K}, \\ &= \alpha \frac{F}{K} + \zeta \frac{Y - F}{K}, \\ &= \zeta \frac{Y}{K} + (\alpha - \zeta)\hat{k}^{\alpha-1}, \\ \frac{K}{Y} &= \frac{\zeta}{1/\beta - 1 + \delta - (\alpha - \zeta)\hat{k}^{\alpha-1}}. \end{aligned}$$

This is (3.3). Equivalently, multiplying by K/Y and using $s^{DM} = Y^{DM}/Y$ gives $(1/\beta - 1 + \delta)K/Y = \alpha(1 - s^{DM}) + \zeta s^{DM}$.

A.7.2 The consumption-equivalent welfare measure

The reference allocation is the stationary equilibrium at zero inflation. Scaling its CM consumption and both DM consumption quantities by $\Delta > 0$, with all effort and search costs held fixed, gives

$$\mathcal{W}_\Delta(0) = \frac{1}{1-\beta}\Big\{\bar{B}\ln(\Delta x) - \bar{A}H - n\mu(\sigma) + n\sigma[\rho(u(\Delta q^c) - c(q^c, K)) + (1-\rho)(u(\Delta q^m) - c(q^m, K))]\Big\}\Big|_{\pi=0}. \tag{A.1}$$

The reported consumption-equivalent cost solves $\mathcal{W}_\Delta(0) = \mathcal{W}(\pi)$ and equals $1 - \Delta$. This scaling is a utility comparison, not a new feasible production allocation. Since consumption utility is increasing, $\mathcal{W}_\Delta(0)$ is increasing in Δ, ensuring uniqueness when a solution exists.

B Additional allocation illustrations

The additional analytical example shows that rising search is insufficient for rising capital. Its derivative numerator remains positive, so the trade-size loss dominates on the selected branch.

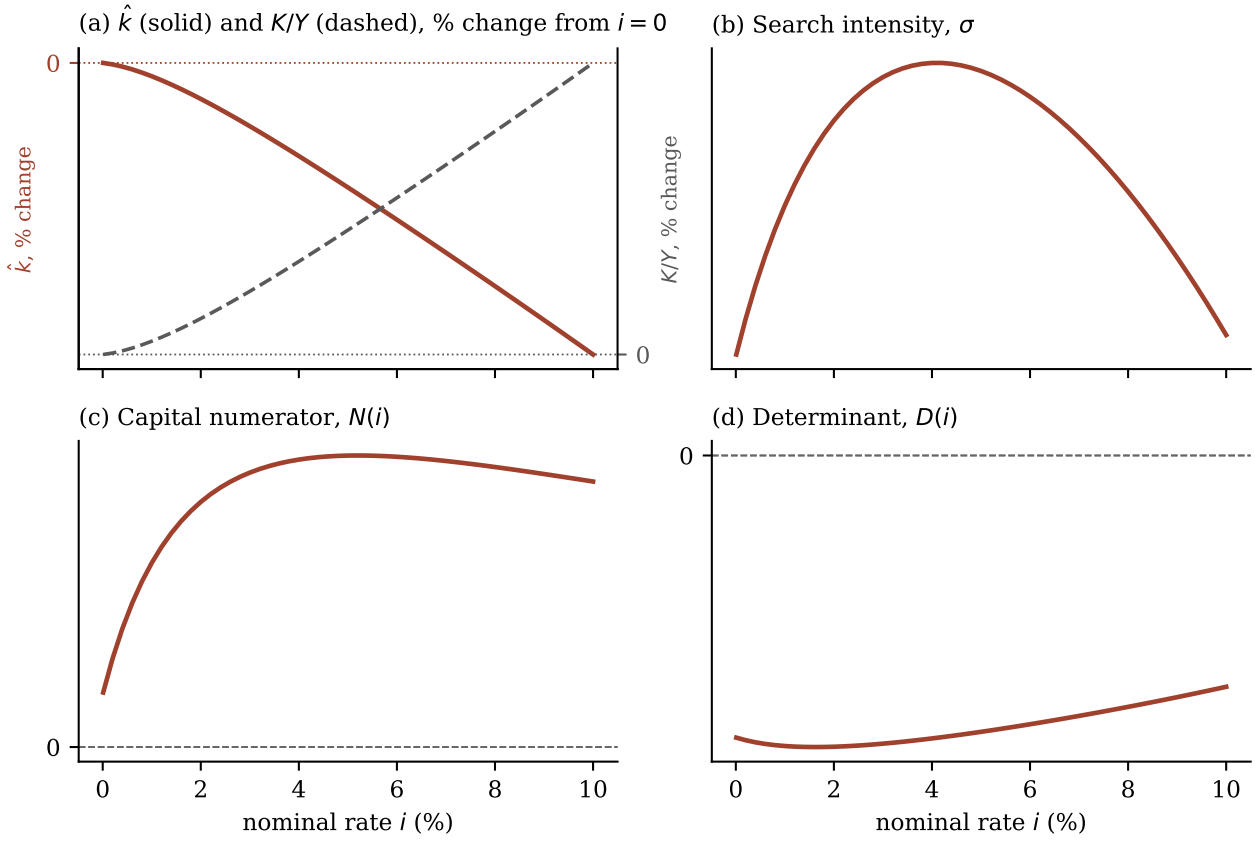


Figure 8: Declining capital with active search. Highest-search equilibrium. Parameters are those of Figure 2, except $\bar{B} = 40$ and $\nu = 1$.

The allocation paths with lower trading frictions show the frequency margin at its upper bound. With every other calibrated parameter fixed, monetary and credit quantities fall as the nominal rate rises, capital declines, and K/Y rises.

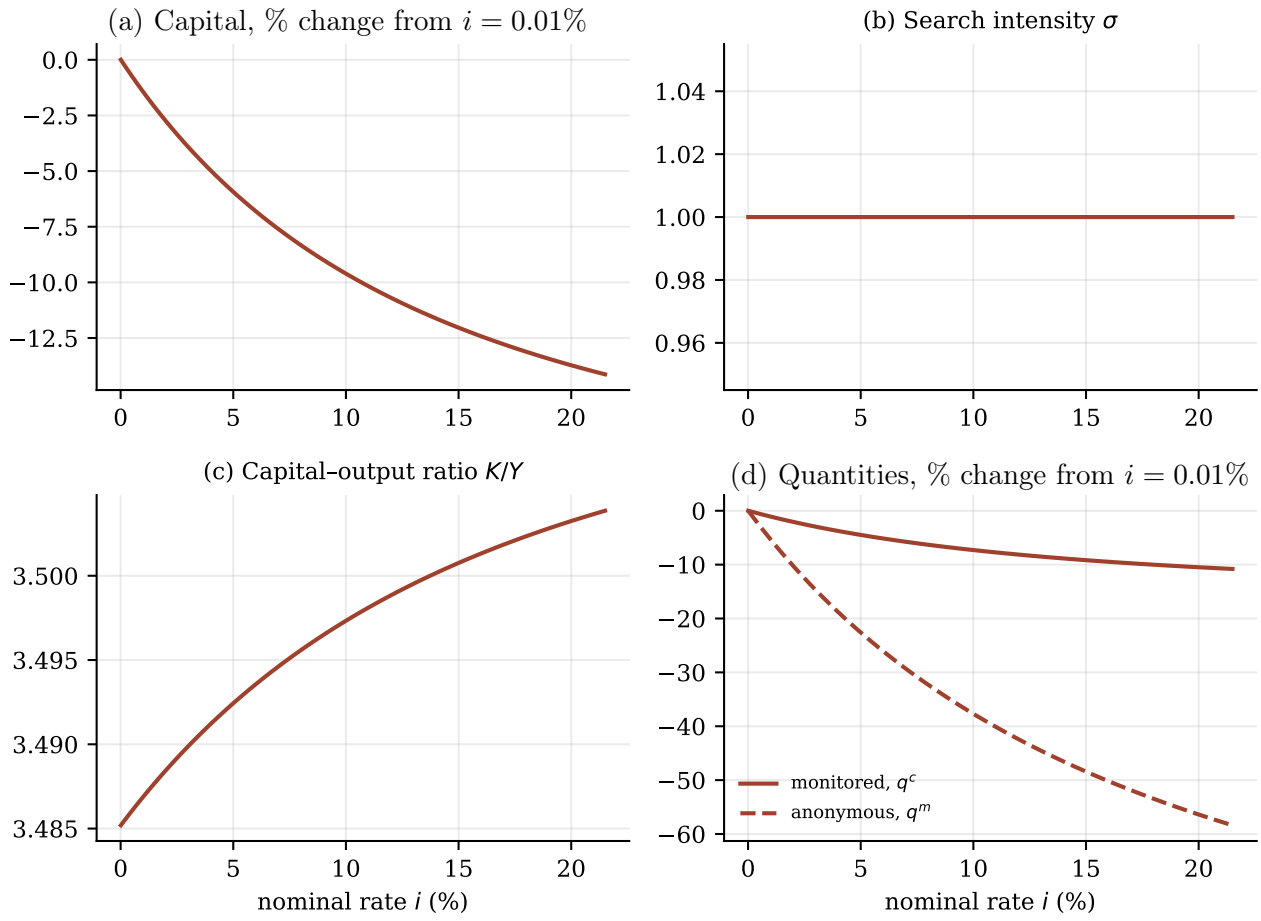


Figure 9: Allocation paths with lower trading frictions: search cost reduced to one fifth. Highest-search branch, with other parameters fixed at the baseline calibration. Search is at its upper bound over the reported range. Panels are as in Figure 4; percentage changes use $i = 0.0001$ (0.01 percent) as the reference.

C Data and descriptive evidence

The data sources cover different periods and different definitions of capital. Table 3 records the coverage of the underlying datasets. Coverage is broader than the eligible samples in individual figures. The international pooled figure additionally restricts inflation to -5 to 20 percent, K/Y to 0.3–15, and requires at least 25 annual observations before averaging. The pooled sample contains 135 countries in the all-country panel, 46 in the U-shaped group, and 21 in the rising group.

Country shape classification is based on smoothed endpoints and minima, not a monotonicity test. The classification script estimates a Gaussian kernel mean with bandwidth 2 percentage points on a grid between the 4th and 96th percentiles of the transformed inflation observations. A U-shaped label

Table 3: Sources and construction of the descriptive evidence

Series	Source	Coverage	Construction
Panel A. Capital–output ratios and inflation			
U.S. capital–output ratio	BEA Fixed Assets Tables, via FRED	1929–2024	Net fixed assets at current cost relative to nominal GDP
International capital–output ratio	Global Macro Database (Müller et al., 2025)	148 countries, 1970–2024	Perpetual-inventory capital from real investment, depreciation 0.07, relative to real GDP
International capital–output ratio	Penn World Table 11.0 (Feenstra et al., 2015)	144 countries, 1970–2023	Capital stock relative to GDP at constant national prices
Long-run capital–output ratio	Jordà–Schularick–Taylor, release 6 (Jordà et al., 2017)	18 countries, 1870–2020	Perpetual-inventory capital, depreciation 0.05, relative to real GDP
Inflation	Global Macro Database; Jordà–Schularick–Taylor for U.S. data through 2020	Corresponding capital samples	Annual consumer-price inflation
Panel B. Payment-access proxies			
Cash machines	World Development Indicators (World Bank, 2025a)	191 economies, 2004–2024	Machines per 100,000 adults; median across years
Bank branches	World Development Indicators	187 economies, 2004–2024	Commercial-bank branches per 100,000 adults; median across years
Account ownership	Global Findex 2025 (World Bank, 2025b)	162 economies, first available wave	Adult share with an account at a financial institution or mobile-money provider
Digital payments	Global Findex 2025	158 economies, first available wave	Adult share making a digital payment

requires an interior minimum below both endpoints. Remaining schedules are labeled rising when the last fitted value exceeds the first. Agreement across the two capital datasets yields 50 U-shaped and 22 rising classifications before the pooled-figure eligibility restrictions. Table 4 lists these groups for transparency.

Table 4: Country groups with agreement across capital datasets

Classification	Countries
U-shaped (50)	Albania, Angola, Argentina, Benin, Burundi, Cameroon, Canada, Congo DR, Czech Republic, Ecuador, Egypt, Equatorial Guinea, Estonia, Guinea, Haiti, Indonesia, Iraq, Ireland, Israel, Ivory Coast, Japan, Jordan, Kuwait, Laos, Latvia, Lithuania, Macedonia, Madagascar, Malawi, Mali, Mauritius, Mexico, Mozambique, New Zealand, Peru, Philippines, Poland, Portugal, Republic of the Congo, Romania, Rwanda, Sierra Leone, Sri Lanka, Sudan, Taiwan, Tanzania, Togo, United Kingdom, Uzbekistan, Vietnam
Rising (22)	Australia, Belgium, Dominican Republic, El Salvador, Eswatini, Georgia, Greece, Guatemala, Iran, Italy, Kazakhstan, Morocco, Namibia, Nigeria, Norway, Oman, Paraguay, Russian Federation, Slovakia, South Africa, Turkmenistan, United States

Notes: Groups require matching classifications across the two capital datasets. The rising classification is an endpoint comparison after excluding U-shaped schedules. These groups are formed using the outcome being illustrated.

Selected examples make the payment-access interpretation visible without providing an independent test. The first two rows of Figure 10 show Burundi, Cameroon, Madagascar, Mozambique, Sierra Leone, and Sudan, selected from the low-cash-machine-access group with U-shaped classifications in both capital datasets. The last two rows show the United States, Australia, Belgium, Italy, Greece, and Slovakia, selected from the high-access group with rising classifications. The selection conditions should be kept in view when interpreting the apparent contrast.

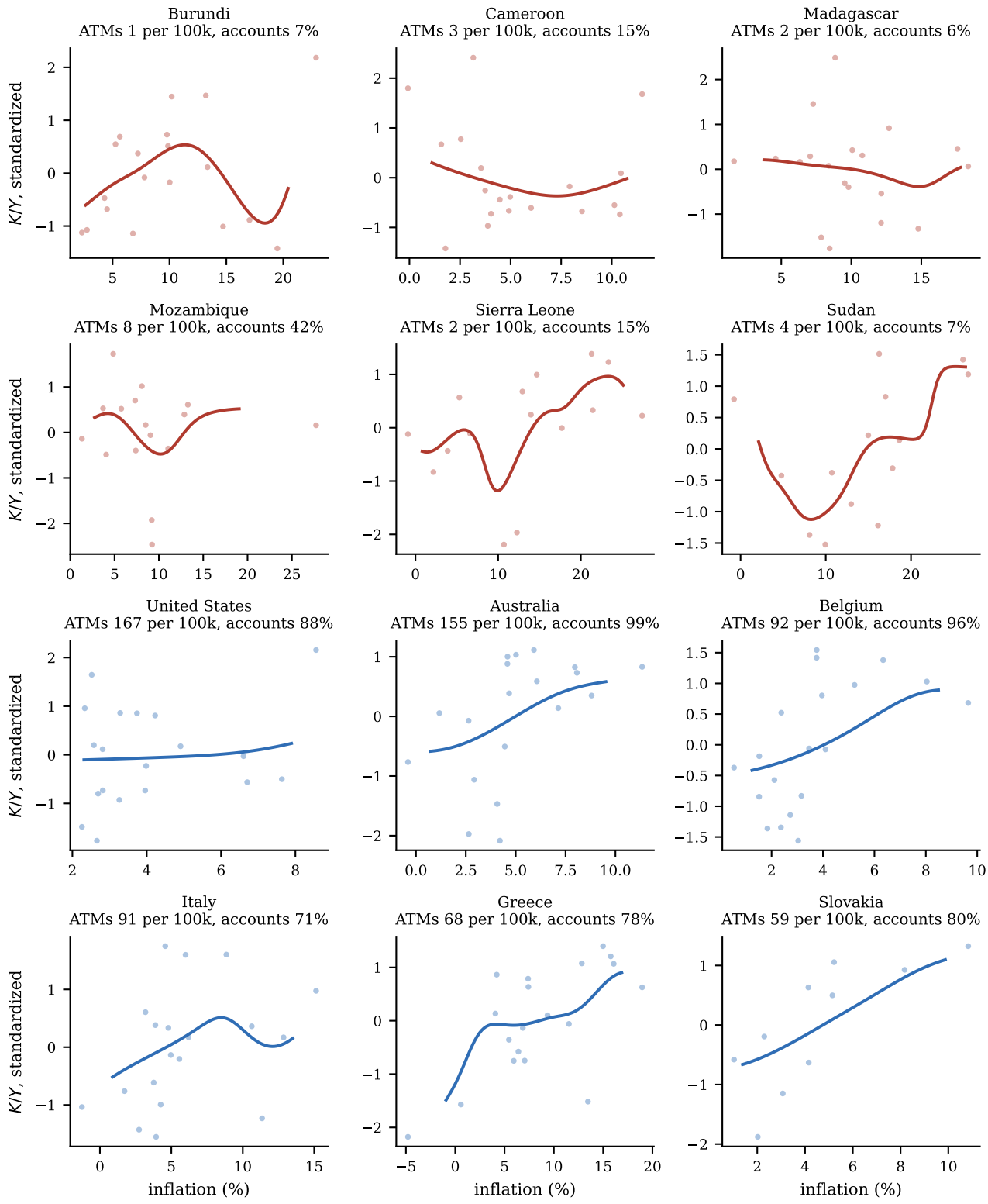


Figure 10: Country examples selected on payment access and schedule shape. Global Macro Database perpetual-inventory capital relative to real GDP, three-year averages, with country-specific linear trends removed and K/Y standardized within country. Points are observations; lines are kernel means. The first two rows select low-access U-shaped countries; the last two select high-access rising countries. These examples are descriptive.